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FINITE ELEMENT MODELING OF TWO-DIMENSIONAL CASCADE FLOWS

Technical Report No. SW-78-1

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University, Mississippi 38677

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This study develops a new numerical solution technique to simulate the transonic cascade flow through turbomachines, and also improves the understanding of the basic characteristics of turbomachine flows through a systematic process of improvements to increase the level of model sophistication. The Finite Element Method (FEM) is especially suitable for solving problems with irregular boundary geometry including curved as well as sharpcornered ones, because the shape and size of each element may be specified arbitrarily and sides of each element may be curved. Therefore, the complication of governing differential equations → next page

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LEVEL II

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I. SUMMARY

Analytic investigations, or more precisely, the computer simulation of flows through a cascade of blades in turbomachines have become more and more important as an essential source of information for the design of high performance turbomachines in recent years, because the pure- and semi-empirical approaches are both costly and time-consuming.

The OBJECTIVE of the proposed study is not only to develop a new numerical solution technique to simulate the transonic cascade flow through turbomachines, but also to improve the understanding of the basic characteristics of turbomachine flows through a systematic process of improvements to increase the level of model sophistication.

The Finite Element Method (FEM) is chosen for its advantages of generality and simplicity of both mathematical formulation and numerical solution. It is especially suitable for solving problems with irregular boundary geometry including curved as well as sharp-cornered ones, because the shape and size of each element may be specified arbitrarily and sides of each element may be curved. Therefore, the complication of governing differential equations through coordinate transformations needed to result a rectangular computation domain in finite difference schemes can be avoided, and the accuracy as well as computing efficiency may be improved by adjusting the size and shape of the elements. Supported by an AFOSR grant (AFOSR-76-2982), the Phase I, Two-dimensional Potential Cascade Flows, and the Phase II, Two-dimensional Viscous Cascade Flows of this project have been completed. Results obtained from the Finite Element Modeling are not only

comparable with those obtained by other numerical schemes, but also sound physically. Since the simulation of these two cases are intended only as stepping stones for studying the model of higher level of sophistication, rather than duplicating the extensive results obtained by various other numerical schemes; the primary contribution of this project is the development of the new modeling technique, based on the FEM.

The success of the FEM in the structure analysis of the turbomachine has been remarkable. Many previously intractable problems in structure design and vibrational analysis have been reduced to routine calculations by using this powerful tool. It is reasonable to believe that the application of FEM in the aerodynamic design of turbomachines shall be successful also in the near future, although there is still a great deal to be done. As the capacity and the speed of modern digital computers continuing to improve, it is expected that the development of three-dimensional flows through turbomachines is highly promising. Therefore, it is highly recommended that the support in this area be increased.

II. A GENERAL DESCRIPTION OF THE PROJECT

The Over-all Objective of the study is to develop a new and efficient technique for turbomachine flow simulation. The FEM is chosen for its advantages of generality and simplicity in both model formulation and solution. It is especially suitable for solving problems with irregular boundary geometry including curved and sharp-cornered ones, because the shape and size of each element may be specified arbitrarily and the sides of an element may be curved. Therefore, the many difficulties due to signalarity or discontinuity are eliminated and the computing efficiency can be improved by simply adjusting the size and shape of the elements as appropriate. Following the basic approach of a systematic process of improvements to increase the level of model sophistication, the over-all objective is to be accomplished in several phases. The First Phase is the development of a system of basic computer subroutines for carrying out every step of formulation as well as solution using the FEM. Although the testing case used in the first phase is the potential cascade flow for the convenience of verification; the computer codes developed are carefully designed so that they may be used, either as they are or after minor modification, as "building blocks" to "construct" programs for simulating flow models at higher levels of sophistication. Results obtained in the Phase I studies are in good agreement with those published elsewhere. Studies for improvement of efficiency of each subroutine as well as the overall system have also been completed. A brief outline of the significant accomplishment of Phase I studies is given in Chapter IV. More details are included in a paper entitled "Computer Simulation of Cascade Flows in Axial-Flow Compressors" and Su's thesis entitled Computer Simulation of Cascade Flows of Ideal Fluids. Copies of these two publications are given in the Appendices of this report.

Although both Viscous Cascade Flows and Compressible Cascade Flows are proposed for simultaneous study in the Phase II of the project; the last minute reduction in support by AFOSR, has forced the investigators to concentrate their effort on the Viscous Case only. Results obtained from the Phase II study is briefly presented both in Chapter V and in an Abstract of a paper entitled Finite Element Modeling of Two-dimensional Viscous Cascade Flows (see Appendix A-1).

If further fundings are available, the investigators would like to continue their contribution in the advancement of the state-of-the-art on Finite Element Method in Turbomachinery Flow Simulation. They have experienced that the FEM is extremely suitable for the highly irregular geometry of turbomachinery flows, that the computer codes developed are quite general and versatile, i.e. one program can be applied to several different cases, and that computing time can be reduced by refining the algorithm. Therefore, they are confident that the success of the FEM application to aerodynamic analysis of the turbomachinery design may match that of the FEM application to the structure and vibration analysis of the turbomachinery design in the near future.

III. LIST OF PUBLICATIONS

The following is a chronological bibliography of publications and significant scientific papers resulting from the work performed under the support of this grant (AFOSR-76-2982):

1. Wang, S.Y.; Mach, K.D. and Su, T.Y.; "Computer Simulation of Cascade Flows in Axial-Flow Compressors" presented at the First International Conference on Applied Numerical Modelling, held at University of Southampton, England, July 11-15, 1977. Published in the book, Applied Numerical Modelling, ed. by C.A. Brebbia, Pentech Press, London, 1978.

2. Wang, S.Y.; "Finite Element Solutions of Transonic Flows in Axial-Flow Turbomachines" Progress Report submitted to US-AFOSR, Jan. 20, 1977.

3. Wang, S.Y.; "Finite Element Solutions of Transonic Flows in Axial-Flow Trubomachines, Phase I: Potential Cascade Flows," Interim Scientific Report submitted to US-AFOSR, July 30, 1977.

4. Su, T.Y.; "Computer Simulation of Two-dimensional Cascade Flows of Ideal Fluids," M.S. Thesis, the University of Mississippi, School of Engineering, May, 1978.

5. Wang, S.Y. and Su, T.Y.; "Finite Element Modeling of Two-dimensional Viscous Cascade Flows," paper in preparation for presentation at and publication in the proceeding of the Second International Conference on Computational Methods in Nonlinear Mechanics, to be held at the University of Texas, Austin, Texas, March 26-30, 1979.

IV. RESULTS OF POTENTIAL CASCADE FLOWS

Using the two-dimensional potential flow through a cascade of airfoils as a testing case, results have been obtained in terms of velocity, stream function, and pressure distribution (Figures 1, 2, and 3). They are not only physically reasonable, but also in good agreement with both analytic solution of the potential flow theory and some experimental data of low-speed turbomachinery flows.

Extensive effort has been devoted in developing computer codes being both accurate and efficient. Since the shape as well as the size of each element affect the accuracy of the F.E. Solution, and the number of elements used in discretization of the flowfield will be related to the computing time required for generating solution; several F.E. systems have been studied. The isoparametric formulation using quadralateral elements (Figure 4) have been found more desirable than using triangular elements (Figure 5). The correlation between the computing time required and the number of quadrilateral elements used in the discretized domain has been found. It seems that the computing time required is roughly proportional to the cubic power of the number of elements used. More importantly, results with good accuracy have been obtained from an element system with relatively small number of elements; although one can always generate more accuracy for a great deal less. It is worth noting that since the FEM is essentially an implicit scheme; therefore, one does not have to worry about the problem of computational instability at all.

Each computer code (subroutine, function, etc.) as well as the computer program has been carefully examined and tested to insure that they all be highly efficient. For example, using the Incident Symbol to assemble local finite element equations into a complete set of global equations is the most convenient way to do the job as far as the mathematics and programming are concerned; unfortunately, it requires a lot of computer storage and computing time, thus, not efficient. A new scheme has been developed by us, which saves both computer storage and, more importantly, reduces the computing time by two-thirds. With this improvement, a potential cascade flow can be completely solved by our computer program within a few minutes. Therefore, we are quite encouraged to attack turbomachinery flow models of higher level of sophistication.

The stream function as well as the velocity potential formulations have also been tried. It has been found that although the velocity potential formulation gives better pressure distribution on the surface of airfoils; however, some difficulties in defining the boundary conditions at the exit of the computing domain (see Figure 4) have been experienced. Fortunately, the uniform velocity assumption seems to give good results. On the other hand, the stream function formulation, has been found being much more convenient than the velocity potential formulation, because the boundary conditions required are not only of the Dirichlet type but also needed only on solid and periodical boundaries, (no boundary conditions are needed at the entrance and exit plans) and its solution is quite accurate in the entire flowfield. This is another advantage of the FEM or the Variational Method in general for that matter.

Of the different element systems attempted, it has been found that the ones we developed (see Figure 4) are superior than the ones used by previous workers (see Figure 5 and 6). The element system developed by us involves less number of elements than those required by the one in Figure 5 to achieve the same accuracy; therefore, it is more efficient. It is better than the one shown in Figure 5 from another viewpoint, that is that global coordinate of each node can be easily generated by computer rather than determined by some measurement and put into the computer as an input data file. Compared with the computation domain shown in Figure 6, it is obvious that ours is more reasonable, because a large portion of fluid in the exit region of the domain in Figure 6 didn't come from the entrance of the domain, so that there are inconveniences in imposing boundary conditions in the case of Figure 6.

It should be emphasized that the Kutta Conditions at the trailing edges of the blades are extremely important to the accuracy of the Finite Element Solutions of potential cascade flows. A few cases of blades with very thin trailing edges have been tested without imposing the Kutta conditions. Results are found to be very sensitive to the thickness of the trailing edges.

A copy of the paper summarizing the approach to solution as well as typical results has been published in a book, APPLIED NUMERICAL MODELLING, edited by C.A. Brebbia, printed by Pentech Press, London is included in the Appendices of this report.

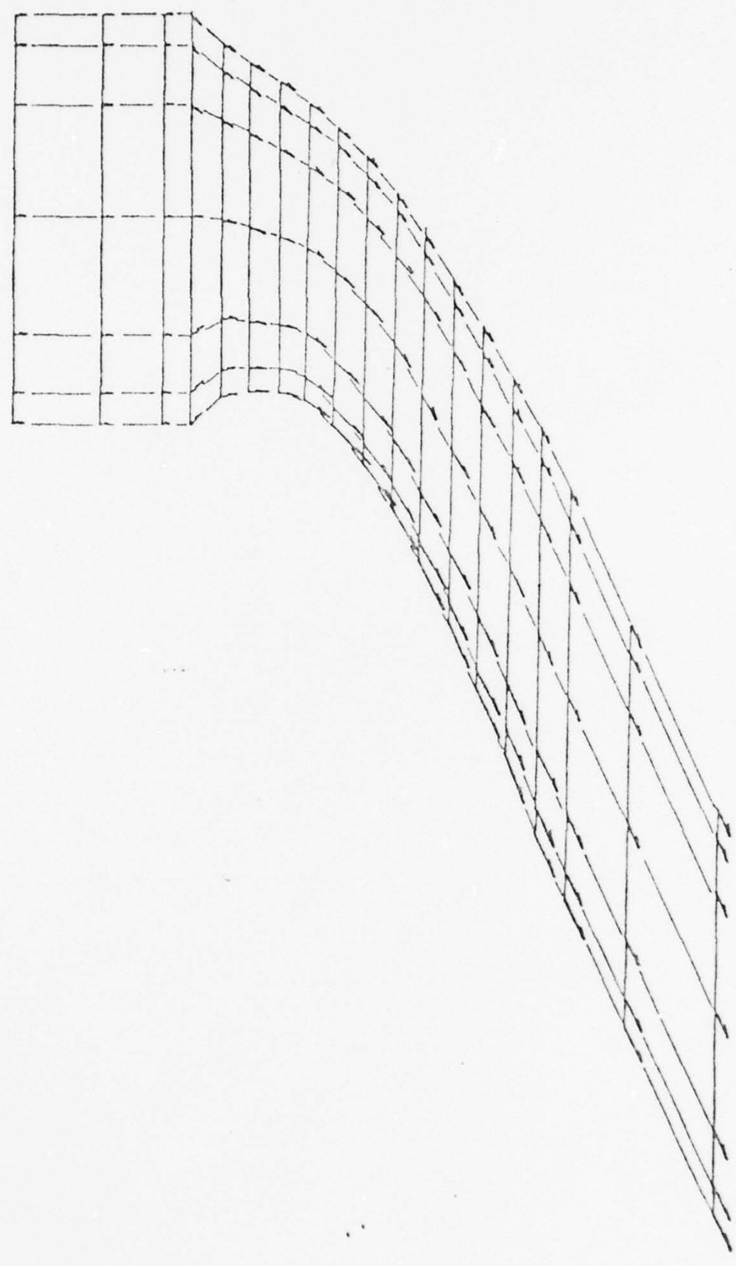

Figure 1-a FE Solution Velocity Distribution with Element System Superimposed

Figure 1-b FE Solution
Velocity Vector Field (Computer Plot)

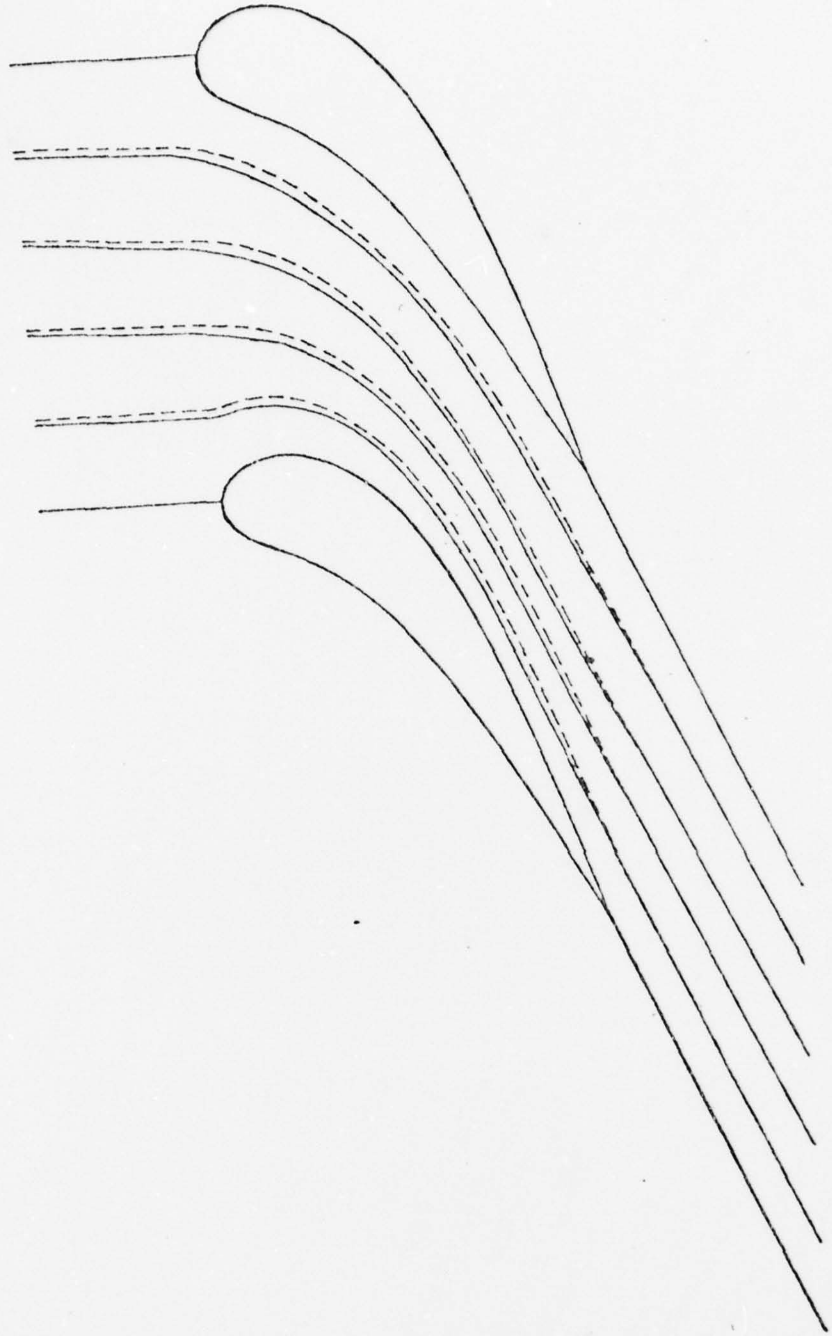

Figure 2 Streamline Plot (FE Solution)

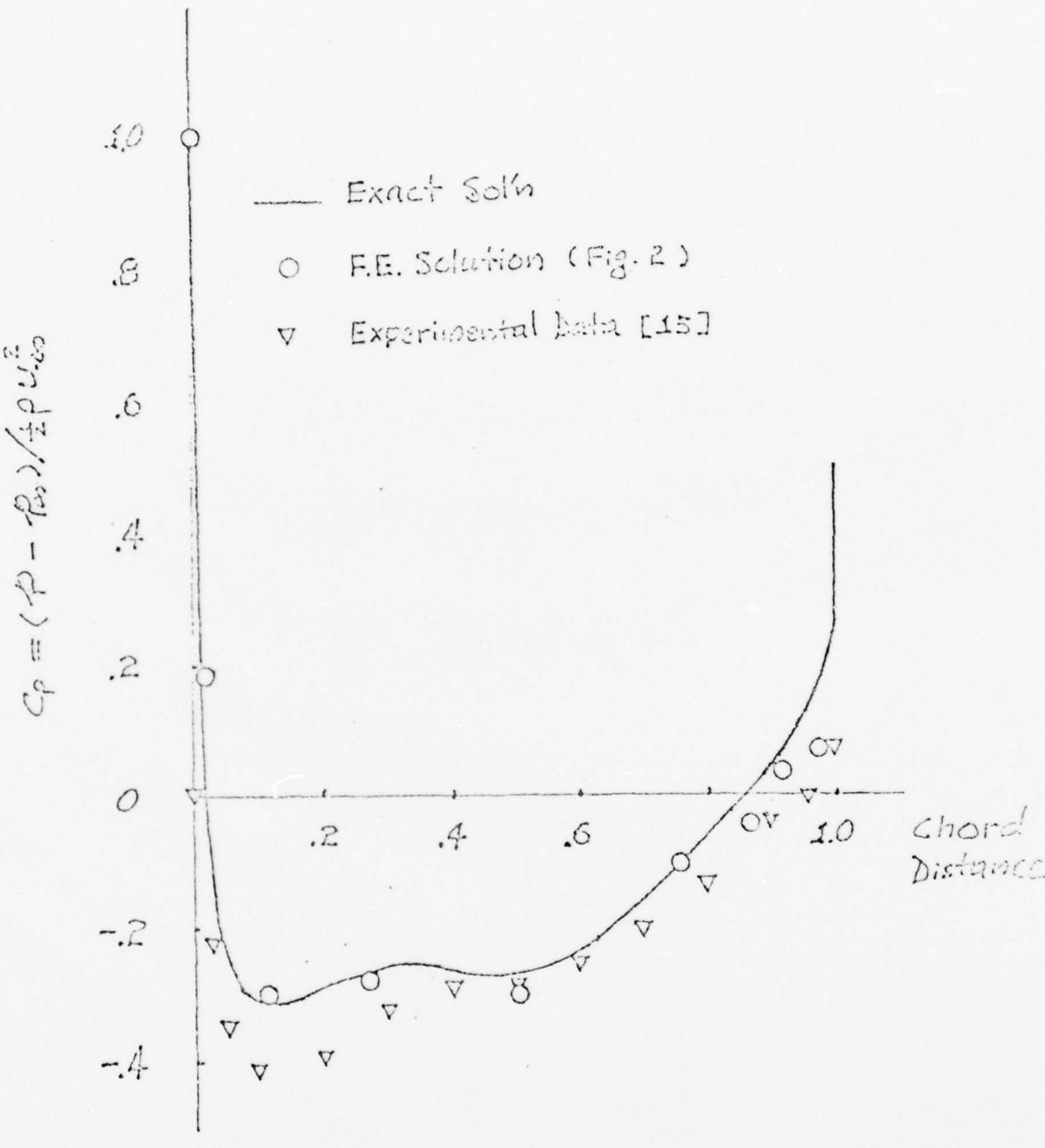

Figure- 3 Pressure Coefficient Distribution of a Symmetric Airfoil

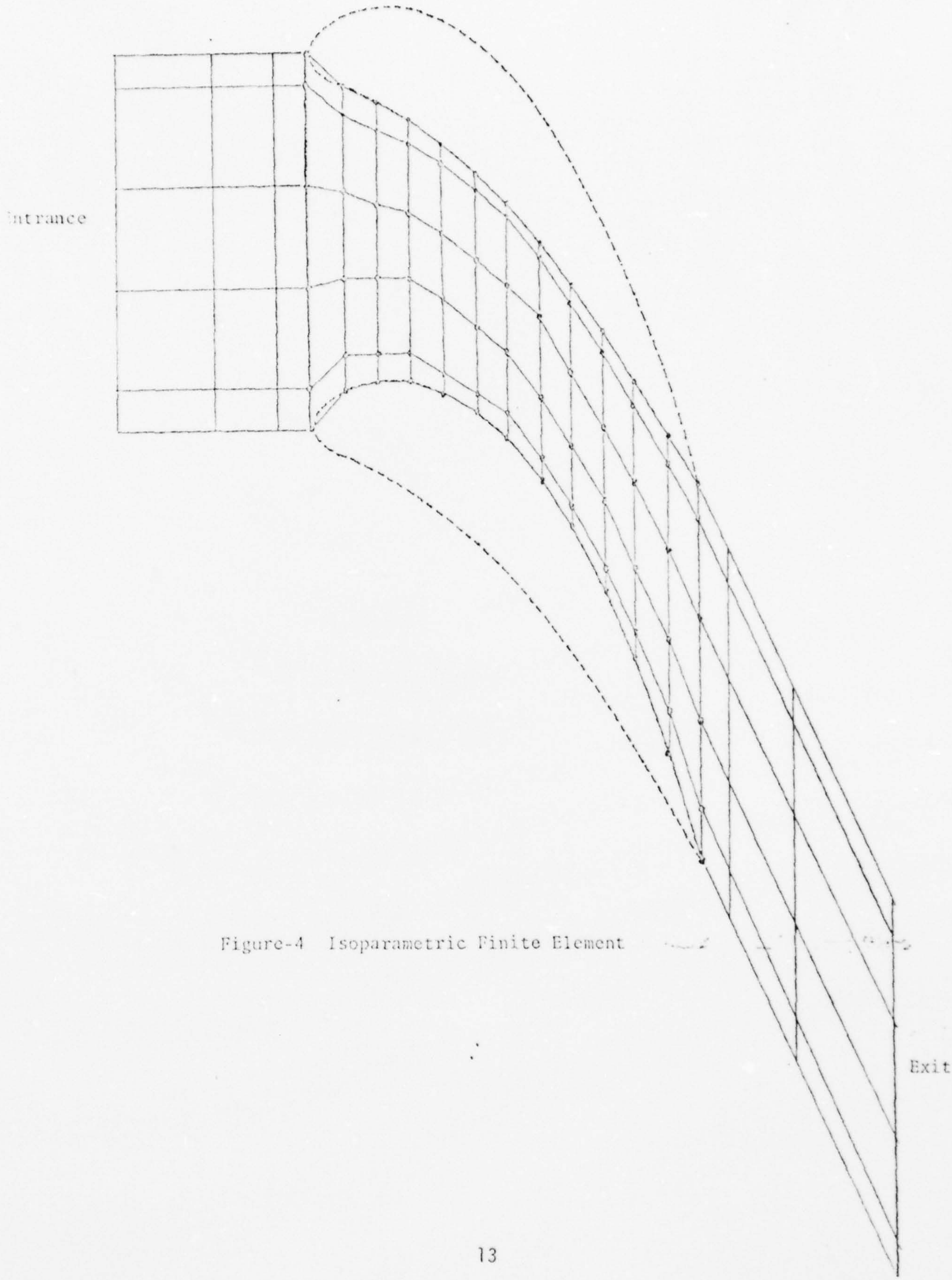

Figure-4 Isoparametric Finite Element

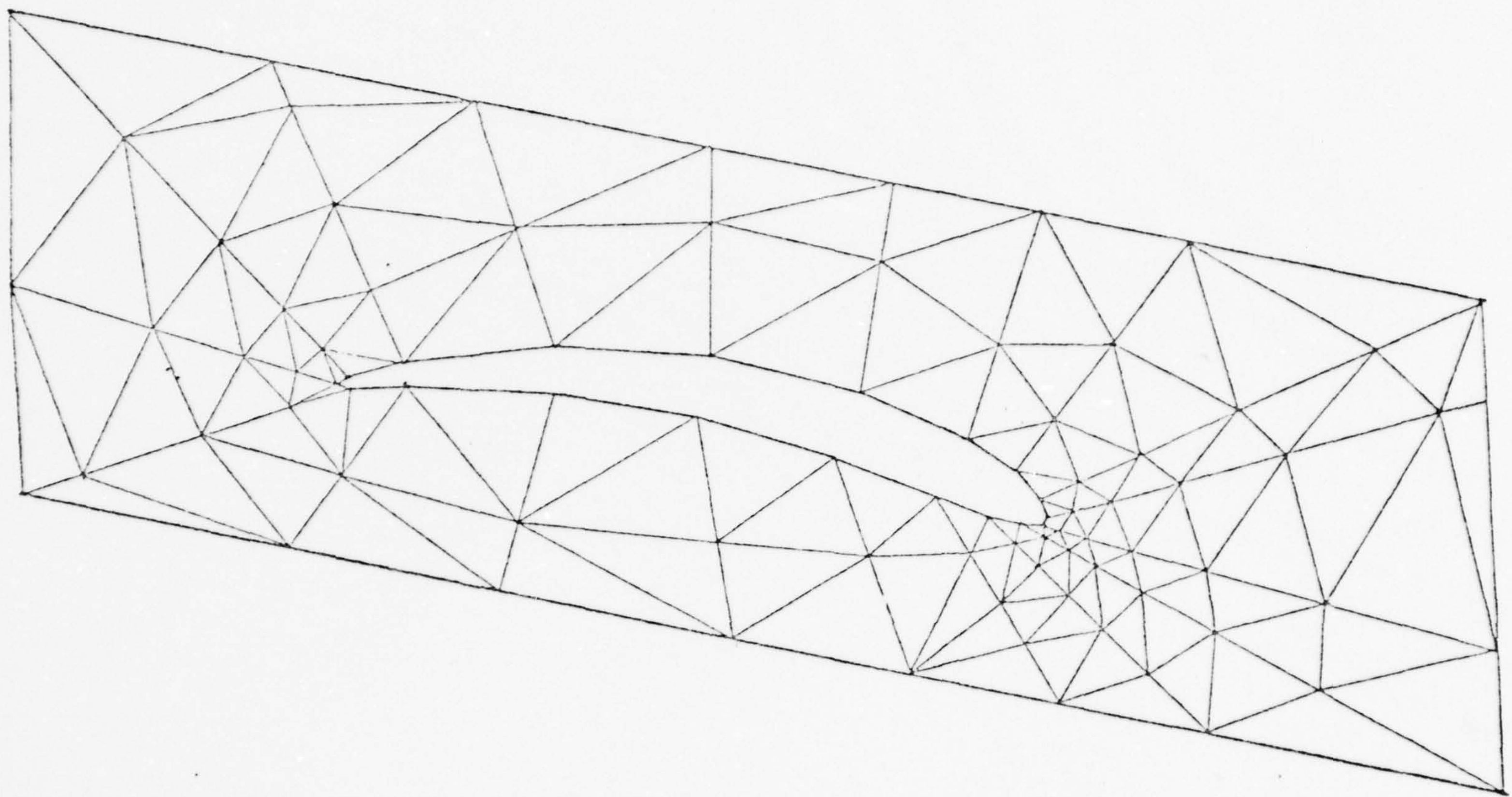

Figure 5 Finite Element System for a Chambered Airfoil

(cf. Thompson, D.S.)

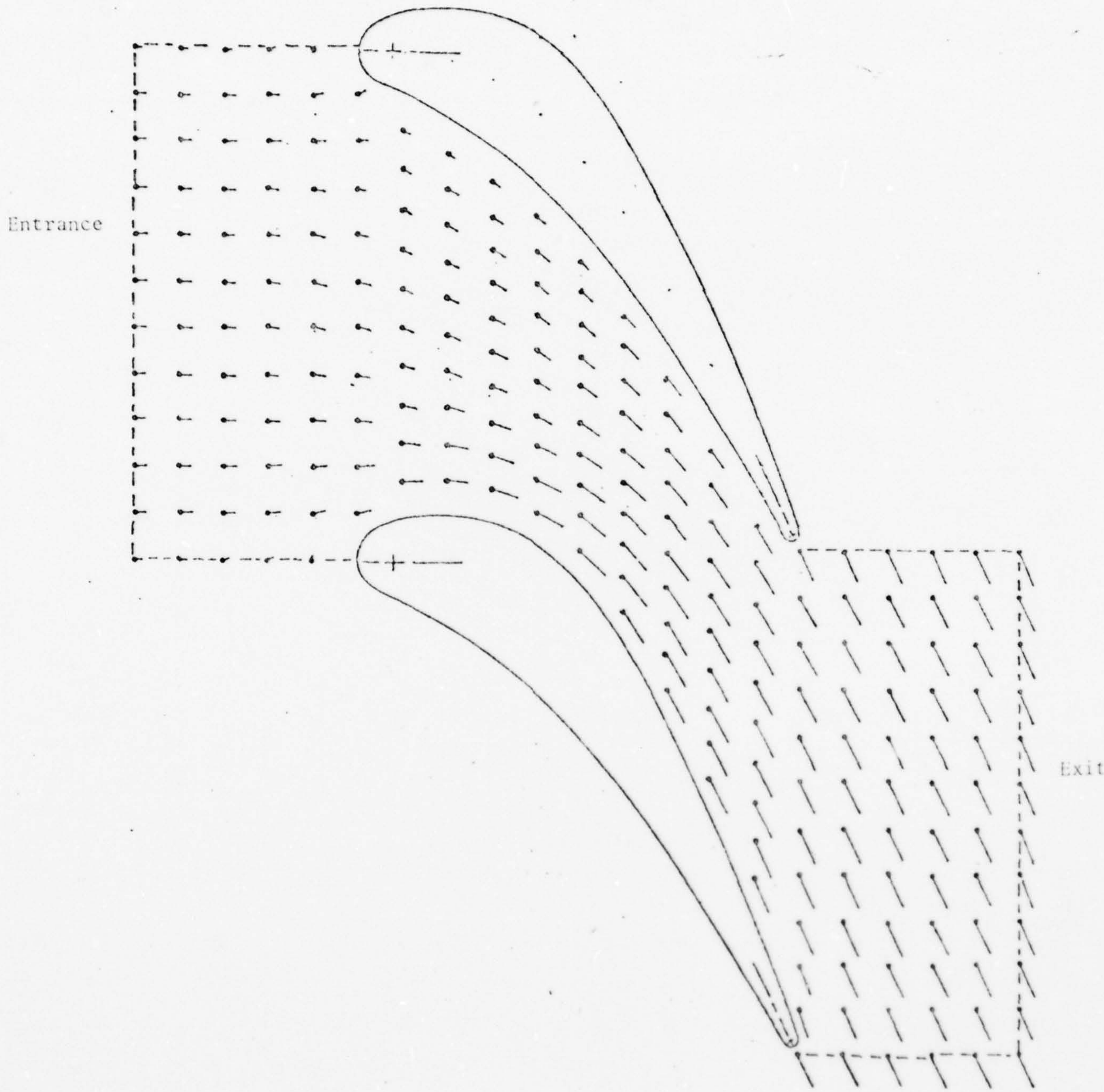

Figure 6 Computation Domain used by R.A. Delaney

V. RESULTS OF VISCOUS CASCADE FLOWS

Due to the relatively narrow passages of the turbomachinery flow and the physical evidences of boundary layer formation, separation and reattachment, etc, the viscous effects appear to be important. Therefore, the Phase II of the project is the Finite Element Modeling of the Two-dimensional, Viscous, Cascade Flows. Following the policy of a systematic approach to increase the level of model sophistication, the low-speed laminar case was considered.

The basic assumptions, mathematical model development using the FEM, solution procedures, and results obtained will be discussed in details in a forthcoming paper entitled, "Finite Element Modeling of Two-dimensional Viscous Cascade Flows", to be submitted for publication in the proceedings of the Second International Conference on Computational Methods in Nonlinear Mechanics. Its final draft will be sent to ASOFR for review. A copy of its Abstract is attached in the Appendices of this report.

The computer plots of a few typical cases are given in figures 7, 8, 9 and 10. The effect of pressure difference between the entrance and exit of the cascade is seen in figure 7 and 8, which shows that the speed is increased when the pressure difference is raised. When angle of attack is changed from 5° to 10°, only minor variation in flowfield is seen in figure 8 and 9. This is probably due to the strong pressure gradient which may have dominanted the flowfield characteristics. The effect of blade shape, the thickness and camber of the blades, is more pronounced. As one can see from figure 10, that there seems to have a separation region existing over the last one third of the suction surface, which is a phenomenon one expects in the viscous flow. Generally speaking, the flowfield properties obtained from the present finite element modeling are physically reasonable and comparable with those of others published.

One special feature added to the F.E. Modeling technique for the two-dimensional viscous flow case is the utilization of the Mixed Interpolation Functions. The first order interpolation functions are used for approximating the velocity field, and the second order interplation function is adopted for approximating the pressure field. This is necessary for achieving the uniform errors in pressure field and velocity feild; because the highest differential operators of pressure and velocity in the Navier-Stokes equations are of different orders. Although more computer storage are required, the results are much better.

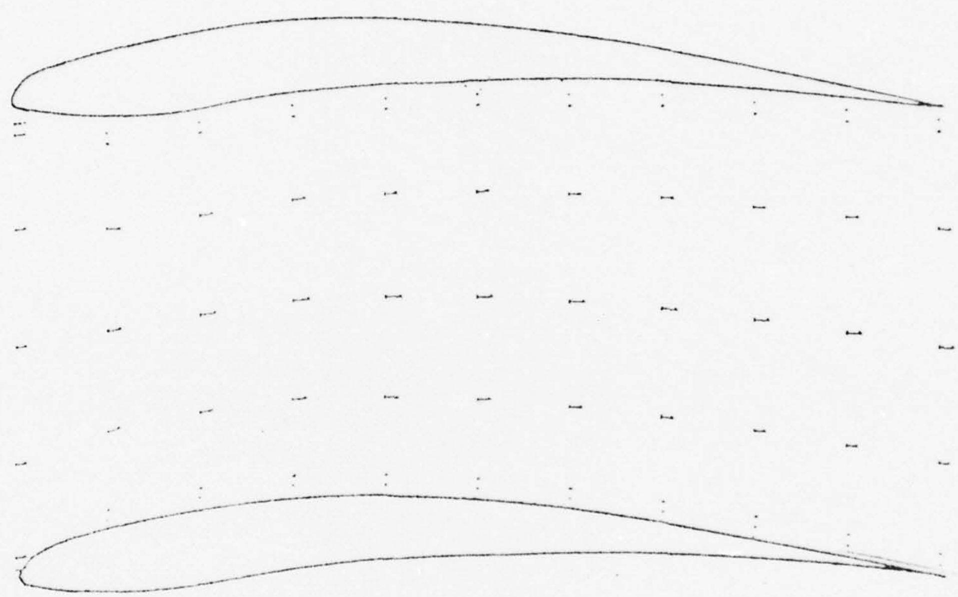

Figure 7. Two-dimensional Viscous Cascade Flow
$\Delta p = 200\ g/cm^2$ and $\alpha = 5°$

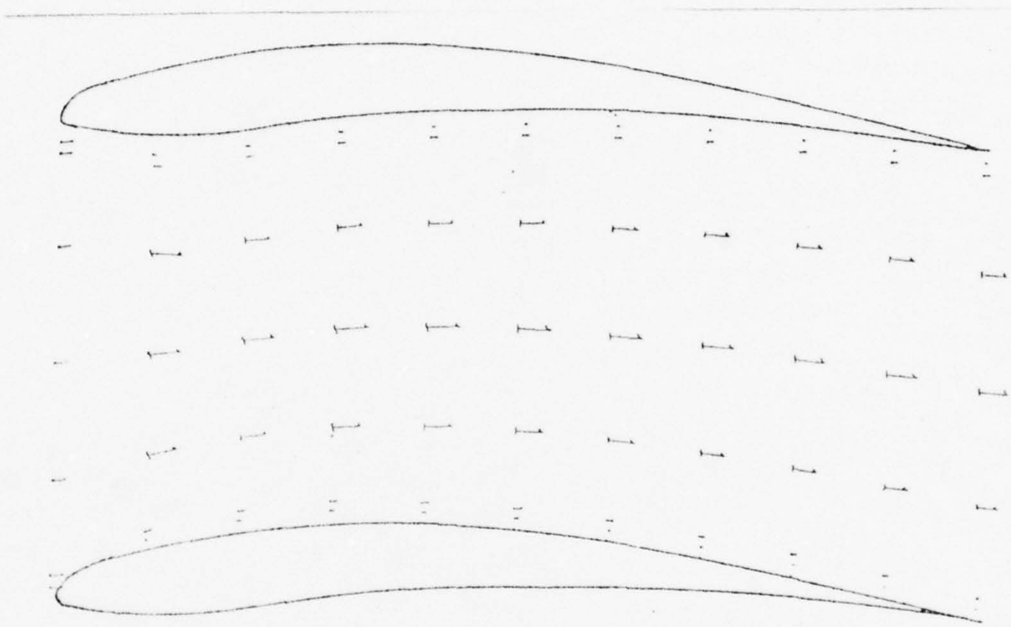

Figure 8. Two-dimensional Viscous Cascade Flow
$\Delta p = 500\ g/cm^2$ and $\alpha = 5°$

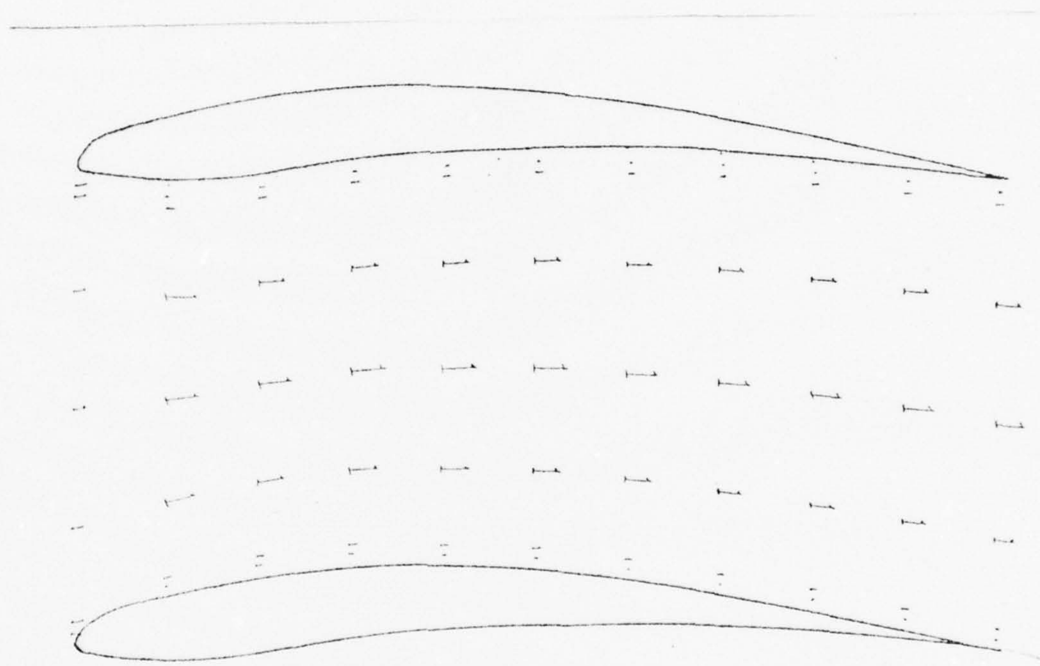

Figure 9. Two-dimensional Viscous Cascade Flow
Δp = 500 g/cm^2 and α =10°

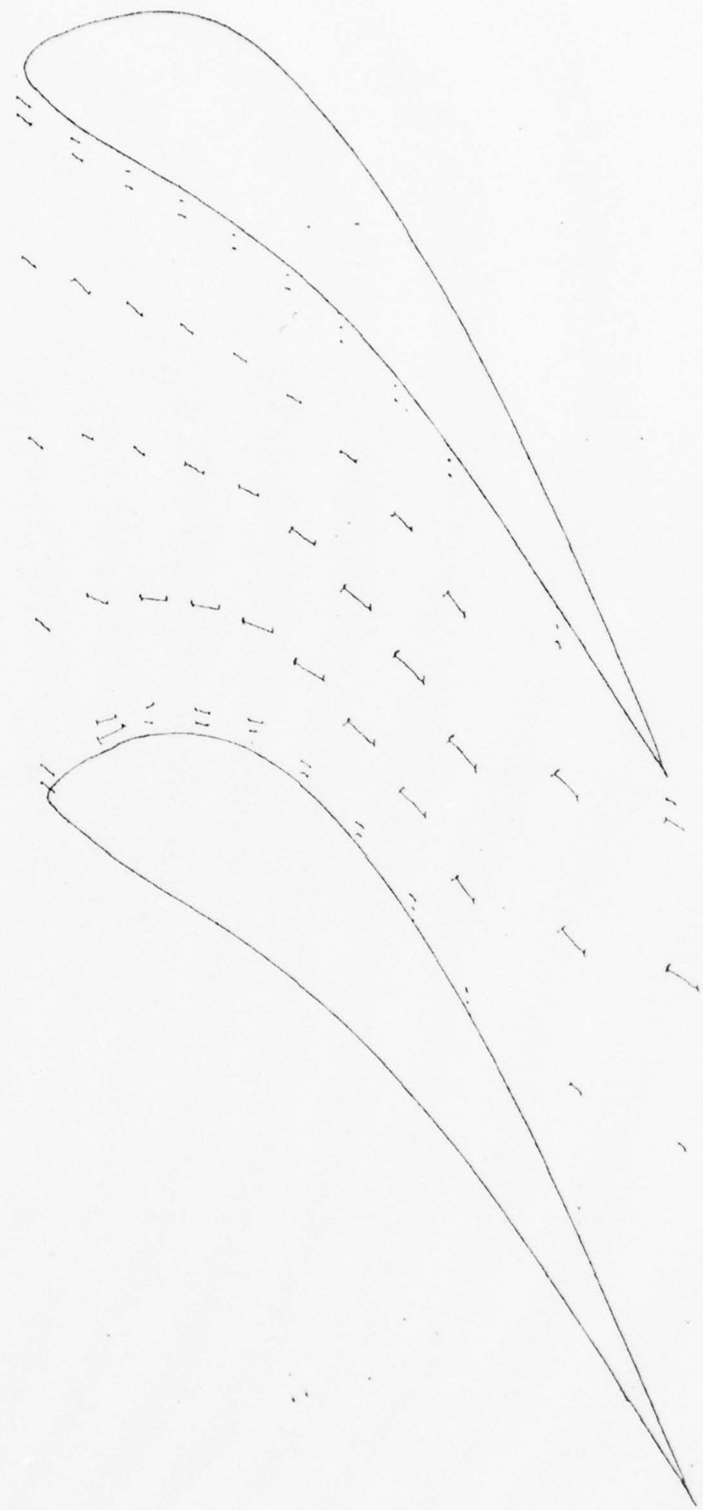

Figure 10. Two-dimensional Viscous Cascade Flow for thick airfoil with higher camber $\Delta p = 500\ g/cm^2$ and $\alpha = 5°$

VI. CONCLUSIONS

Finite element models of the steady, two-dimensional, potential, as well as viscous cascade flows have been developed. Numerical results for various cases have been generated using a Fortran IV Computer Code including several subroutines, functions, as well as a main program. Results of typical cases, such as ones plotted in figures 1, 2, 3, 7, 8, 9, 10 conform well with the potential and viscous cascade solutions of others. By examining the velocity vector plot (figure 1), one can see that the magnitude of the velocity increases where the cross-section area of the flow path decreases and vice versa. The direction of the velocity at every node on the pressure and suction surfaces has been calculated and found tangent to the blade surface in the case of potential flow model.

To further verify the correctness of the Finite Element Solution, the normalized static pressure distribution along the pressure and suction surfaces of the cascade flow is reduced from the stream function solution and plotted Figure 3 to compare with the Delaney's* result based on the highly sophisticated finite difference model as well as some experimental data of hte same cascade configuration provided in Delaney's dissertation. The deviation between the finite element solutions and Delancy's result is primarily due to the difference in the basic model assumptions. The present model is based on the incompressible fluid assumption; whereas Delaney's model was

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*References are listed only in Su's thesis (See Appendix A-3) to avoid duplication.

based on a subsonic compressible flow assumption. The primary reason for comparing with Delancy's results is the fact that his cascade geometry is adopted for the present study. In the same figure, one can also see the fact that the pressure distribution reduced from the velocity potential solution is in better agreement with Delaney's results than that from the stream function solution. It is generally true that the velocity potential solution is more accurate than the stream function solution. However, the stream function formulation is more convenient in specifying the boundary conditions. During the present study, the Kutta condition at the trailing edge has been carefully satisfied. It has been observed numerically that without Kutta condition a slight deviation in trailing edge thickness of a very thin trailing edge results a large change in flowfield properties, especially the static pressure distribution. This observation demonstrates the importance of the Kutta condition at the trailing edges.

Results of two-dimensional, viscous cascade flows are presented in figures 7 through 10. The effects of pressure gradient across the entrance and exist, the angle of attack, and the blade shape have been investigated. Some typical results are plotted by the computer (see figures mentioned above). In gneeral, the pressure gradient varies the magnitude of velocity away from the boundary layer of the blades, the angle of attack at the entrance primarily affects the direction of the velocity vectors, and the flow tends to separate when the camber and thickness of the blades are increased. More detailed discussion on the viscous case will be given in a paper, "Finite Element Modelling of Two-dimensional, Viscous, Cascade Flow".

Its abstract is included, however, in the Appendices (A-1).

It should be emphasized again that the PRIMARY OBJECTIVE of the present study is to develop a general computer code based on the finite element method for solving the two-dimensional cascade flows. The potential flow model is chosen to serve as a test case for verification purposes in the Phase I studies of this project. Based on the aforementioned results, it is obvious that the computer code is satisfactory and can generate reasonable flowfield characteristics from a relatively simple element system. Some effort has been devoted in improving the efficiency of the finite element model. Although an analytic way to determine the optimum configuration (size-shape) of the finite element system for obtaining a reasonable accurate result at least computing cost has not yet been established; the relation between computing time and number of elements has been established based on the computer experimentation of several different finite-element system configurations. The computing time is approximately proportional to the cubic power of the total number of elements used to discretize the flowfield. Using this result one can, at least, estimate how much more he has to pay for the better accuracy he gets by increasing the number (reducing the size) of elements. Using a special assembling technique developed by the investigators for obtaining the global equation computing time has reduced. Although this special (or improved) assembling technique is not as general as the incidence matrix scheme; it is definitely more attractive for developing a production program. The basic concept of this special assembling technique, and the details of this scheme are described in Su's thesis in Appendix A-3.

One should also note that almost all subroutines and functions developed from the present study are in a form as general as possible, in order that they can be used in the future to simulate cascade flows of higher level sophistication with little or no modification.

It is generally agreed to the fact that the computing time for simulating a typical boundary value problem with moderately irregular boundary geometry by the finite difference and finite element methods are comparable at the present state-of-the-art. In view of the fact that the finite difference schemes have been in existance much longer and thus have much more refinements having been built in to them than that of the finite element method, which is still in its infant stage, as far as flowfield analysis is concerned. Thus, it is quite clear that the finite element modeling of the cascade flow is more promising in the future, because there is plenty of room for refinement.

VII. RECOMMENDATION

It is quite obvious that the realistic turbomachinery flow is a three-dimensional, unsteady, non-uniform, and turbulent flow of a viscous, compressible, and heat-conducting fluid with additional complications of boundary layer separation, cavitation and even shock wave interactions for some cases. Although the computer programs developed from the present study are for the simpliest cases the two-dimensional, potential as well as laminar viscous cascade flows, some basic subroutines are directly applicable to more general and sophisticated models as well, and some other subroutines require only minor modifications. With this basic and quite general computer codes in hand, one shouldn't have to much difficulty to attack the cascade flows of higher level sophistication. Therefore, a major contribution to the application of FEM to the aerodynamic analysis of turbomachines has been made.

As mentioned previously, the application of FEM in turbomachinery flow simulation, or in any fluid flow studies, is relatively new. Many computational fluid dynamists are still refering its development being in the stage of infancy. It is quite true the much remains to be done in order to refine the FEM for flow analysis to the stage that it can compete with the well-developed FDM for the same purpose. It is, however, also true that the application of FDM to simulating turbomachinery flows has met with the difficulty of limits of both computer storage and computing time, as well as the difficulty to deal with the irregular geometry. It may be wise to develop an entirely new apptoach, e.g. the FEM for simulating the turbomachinery flows. And, from the currently existing results for geometrically simple flow problems the computer storage as well as computing time required by FDM and FEM are almost comparable. Remembering that the comparison is made between a well-developed Method (FDM) and a very crude method (FEM), one should believe that with refinements the FEM is likely to show its effectiveness in flow-field silumation especially for flows with irregular geometry. Therefore, it is highly recommended that the Finite Element Modeling of three-dimensional, potential, viscous and transonic flows through turbomachinery be strongly encouraged.

APPENDICES

FINITE ELEMENT MODELING OF

TWO-DIMENSIONAL VISCOUS CASCADE FLOWS

ABSTRACT

A new approach based on the Finite Element Method (FEM) to the simulation of the cascade (blade-to-blade) flows through an axial-flow turbomachines has been developed. Using the method of weighted residual the variational functionals are formulated from the nonlinear Navier-Stokes equations and the continuity equation for the case of two-dimensional, laminar flow of viscous and imcompressible fluids. The rather irregular flow regime of the turbomachinery cascade can be discretized into a set of well-designed two-dimensional, quadrilateral elements with the size and shape of each element chosen for the optimum computing efficiency, i.e. requiring least computing time under the constraint of a certain accuracy. The mixed isoparametric interpolation functions have been adopted for approximating the field properties with the first order interpolation for pressure field and the second order interpolation for the velocity field, because the highest differential operators of these two fields are of the first and second order. The utilization of the mixed interpolation functions gives more uniform errors in these two field quantities, and thus, better uniform accuracy. The discretization in time-domain is accomplished by the method of finite difference. Therefore, the present approach may be referred to as a finite element and finite difference

technique.

The mathematical formulation steps, such as: deriving the finite element equations, assembling into the global set, imposing boundary and initial conditions, and rearranging the final global set to reduce the computer storage requirement and improve the computing efficiency, are carried out by a series of computer sub-programs. Each is designed for a specific step. They are written in a form as general as possible, so that they will be used as building blocks to construct computer codes for modeling turbomachinery flows of higher level of sophistication.

Numerical results for various pressure gradient (between the entrance and the exit), the angle of attack, and the camber and thickness of the blades have been obtained. At relatively high pressure gradient, the flowfield properties are almost dominated by the pressure gradient, so that the effect of angle of attack is of minor importance. The evidence of boundary layer separation is seen near the trailing edge over the suction surface of the blade, which is expected for the viscous shear flow. More extensive numerical results and computer plots will be presented and discussed in the paper.

It has found that this new approach is very versatile with respect to geometrical conditions (like curved and sharp-cornered boundaries) which can be handled in a straightforward way. The computer storage and computing time requirements have been comparable to the well-established finite difference schemes for simulating similar problems. With further refinement the FEM offers a promising alternative to the turbomachinery flow simulation.

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COMPUTER SIMULATION OF CASCADE FLOWS IN AXIAL-FLOW COMPRESSORS

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INTRODUCTION

A better understanding of the basic characteristics of the flow through turbomachines is of increasing importance to both experimental researchers and designers of turbomachinery. It assists the experimentalists in selecting test parameters wisely, and provides the designer with the preliminary design information. As the performance as well as the efficiency requirements have become more and more demanding, the turbomachines have to be operated at much heavier aerodynamic loads as well as much higher inlet speed or temperature. The real flows through turbomachine have become extremely complex phenomena. They are truly unsteady, non-uniform, rotational and turbulent flows of viscous, compressible, and heat-conducting fluids further complicated by the boundary layer separation, cavitation, and reattachment effects as well as the shock wave interactions. The realistic mathematical model includes a set of non-linear partial differential and algebraic equations with very complicated boundary geometry and conditions. Therefore, analytic solutions of the closed form of the realistic model have not been possible, and the classical, much over-simplified analytic solutions have been found inadequate for the turbomachinery design. Traditionally, the development of turbomachines has been relied primarily on empirical approaches. The requirement of the increasingly wide variety of operating conditions and blade configurations of today's high performance turbomachines has made the traditional parametric investigations of pure-as well as semi-empirical nature too costly and time-consuming. Furthermore, the cost of failure of a new turbomachine design during the development phase has reached staggering proportions. Not only will it require millions of dollars to redesign, manufacture, and test its replacement, but the lost of time, which amounts to a year or more, can seriously impede the development cycle. Consequently, the numerical experiments, or computer simulations, of flows in turbomachines have become more and more important as an essential source of information to guide the preliminary design of turbomachines.

Most approximate methods have adopted the approach of separating the turbomachinery flow into two inter-relating components, the blade-to-blade or cascade flow and the meridional plane flow. Circumferential averaging of the equation of motion across the blade pitch yields equivalent axial-symmetric problem solvable in a single representative meridional plane. Information obtained from solutions in blade-to-blade planes must be used in the meridional solution and vice versa. A comprehensive and current review on numerical turbomachinery analysis is given by Japikse (1976). Due to limited space this paper can only cite a few contributions closely related to blade-to-blade flows.

The widely used approach in blade-to-blade analysis is the streamline curvature method, eg. Katsansis (1968, 1975). Kurzrock-Novick (1973) had extended this method to include limited effects of viscosity and compressibility in a time-dependent formulation. This approach usually provides some preliminary design information; however, the model is still far from realistic. More recently, numerous finite difference solutions mostly two-dimensional have been published, such as Davis-Millar (1972), and White-Kline (1975). Fully three-dimensional flow simulations are highly desirable for obtaining fluid dynamic insights of turbomachinery flows. Several approximate techniques for carrying out three-dimensional finite difference solutions, eg. Senoo-Nakase (1971), Bosman-El-Shaarawi (1976), are mostly at the initial stage of a truly three-dimensional viscous turbomachinery flow. Generally speaking, the finite difference method is not desirable in an irregular domain. In order to achieve tractable boundary conditions, some coordinate mapping or even mappings are required, which complicates the governing differential equations in the transformed or computation frame. In addition, the three-dimensional solutions of cascade flow require very large computer storage as well as computing time. The senior authors, Wang (1975) and Mach (1976), of this paper attempted to improve the efficiency of a computer code of Kurzrock's for solving 2-d viscous and compressible cascade flows and found all drawbacks mentioned above. Besides, its many simplifying assumptions, such as locations of stream surfaces being known a priori without successive corrections, variations of properties normal to stream surfaces being ignored, etc. are far from being sound. And, unfortunately, the improvement of

this model requires not only a major effort, if not impossible, but also substantial increase in computing time. Therefore, it seems necessary to look for new alternatives.

Due to the fact that the basic concept of the Finite Element Method (FEM) is simple and extremely suitable for complex geometry, it is chosen to attack the problems of turbomachinery flows. Incidentially, the Finite Element Method is not only nothing new, but considered well-established in the turbomachinery world as a tool of structural designers. Many previously considered intractable problems in structural design and vibrational analysis have been reduced to routine calculations by using this powerful tool. In comparison with such spectacular success, exploitation and application of this tool by aerodynamic designers have lagged far behind.

Recently, the effort in the application of finite element methods to problems of fluid flow has been increased drastically. Again, for briety, only references immediately related are cited. The potential flows around solid bodies were treated by Norrie-De Vries (1971). Gelder (1971) studied the linearized compressible flows. Thompson (1974) reported the finite element solutions of potential flows through a cascade of airfoils. More recently, Hirsch-Warzee (1976) obtained the solution of the meridional through-flow in an axial-flow machine using the FEM. They also proved that not only the results are satisfactory, but also the method (FEM) is simpler and more versatile than other methods existing today.

This paper is presenting the work completed in the Phase I of a research project supported by the USAF Office of Scientific Research with an over-all objective of simulating the three-dimensional, viscous and compressible flows through turbomachines numerically using the finite element method. The over-all objective is to be achieved through a systematic process of improvements to increase the level of model sophistication. Therefore, the immediate objective for the Phase I of the project is primarily the development of computer codes for model formulation and solution using the FEM.

MATHEMATICAL MODEL

In order to develop a numerical solution scheme, which can achieve the over-all objective of simulating a realistic, three-dimensional turbomachinery flow of viscous and compressible fluids, the governing differential equations in tensor notations are:

Continuity, $$\frac{\partial \rho}{\partial t} + (\rho v_i)_{,i} = 0 \qquad 1$$

Momentum,

$$\frac{Dv_i}{Dt} = \frac{1}{\rho} p_{,i} + F_i + \tau_{ij,j}$$

$$\tau_{ij} = (k - \tfrac{2}{3}\mu)\, v_{1,1}\, \delta_{ij} + \mu\, v_{i,j} \qquad 2$$

$$\frac{D()}{Dt} = \frac{\partial()}{\partial t} + v_j()_{,j}$$

Energy,

$$\rho \frac{DE}{Dt} = -\, p v_{i,i} + (KT_{,i})_{,i} + \tau_{ij}\, v_{i,j} \qquad 3$$

State,

$$p = \rho RT \qquad 4$$

$$E = \int c_v\, dT \qquad 5$$

where p is pressure, ρ is density, T is temperature, R is gas constant, E is internal energy, c_v is constant volume specific heat, K is heat conductivity, k and μ are bulk and shear viscosity respectively, δ_{ij} is the Kronecker delta. Equations 1-5 constitute the complete set of equations for unknowns of v_i, p, ρ, T and E. For more general case, C_v, K, k and μ are functions of thermodynamic properties, therefore the state equations determining these quantities should be added. Although the equations listed above are short and neat; unfortunately, they can not be used directly as they are in the modeling of physical problems. The actual equations used are of the expanded scalar form for a particular coordinate system chosen for the problem. The expansion process, especially for some curvilinear coordinate system, such as ones suitable for turbomachinery flows, can be quite involved. Not only the monotonous algebraic manipulations required for the expansion process are tedious and time-consuming; but the unitentional human errors during these manipulations are almost inevitable. In order to eliminate these drawbacks, computer programs using symbolic computing languages, such as FORMAC, MATHLAB, and REDUCE, have been developed. It turned out that these programs can not only expand the conservation equations 1-3 into their scalar form, but also simplify them according to selected assumptions. With this facility, the final set of equations, expanded in scalar form for any curvilinear, orthogonal coordinate system as well as simplified according to pre-selected assumptions, can be generated by digital computers in a matter of minutes without human errors. For example, to derive the governing differential equations for a steady two-dimensional cascade flow of an ideal fluid in the Cartesian coordinate system, one has to specify the scale factors of the coordinate system and the assumptions of time-independency, two-dimensionality, constant density, zero viscosity and body force. The output from the computer will be the simplified continuity and momentum equations. If one wishes to use the stream function ψ or velocity potential ϕ formulation, he may further

define

$$v_1 = \frac{\partial\phi}{\partial x_1} \text{ or } \frac{\partial\Psi}{\partial x_2} \qquad 6$$

$$v_2 = \frac{\partial\phi}{\partial x_2} \text{ or } -\frac{\partial\Psi}{\partial x_1}\,. \qquad 7$$

By a simple substitution, the computer will give the resulting equation in the form:

$$OH = \frac{D^2 H}{D X^2} + \frac{D^2 H}{D Y^2} \qquad 8$$

where H may be either Ψ or ϕ, OH designates zero numerically, and D/DX represents partial derivative. This is equivalent to the Laplace equations of

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad 9$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad 10$$

The boundary conditions of 2-d cascade flow (see figure 1) are:

$$\nabla\Psi\cdot\hat{n} = 0 \text{ or } \phi = \phi_B \text{ on } C_1 \text{ (extrance and exit)} \qquad 11$$

$$\Psi = \Psi_B \text{ or } \nabla\phi\cdot\hat{n} = 0 \text{ on } C_2. \text{ (solid wall)} \qquad 12$$

where $\hat{n}$ is the unit normal vector on the boundary.

FINITE ELEMENT SOLUTION

The finite-element method is a systematic procedure through which continuous function is approximated by a discrete model consisting of a set of values of the given function at a finite number of points in its domain together with piecewise approximations of the function over a finite number of sub-domains. These subdomains are called *finite elements*, and the local approximation of the function over each finite element is uniquely defined in terms of the discrete values of the fnction at the finite number of preselected points (nodes) in its domain. The finite element solution procedure is outlined below using Laplace equation as an example. First, the entire flowfield (to be referred to as the global system) is divided into an appropriate set of finite element subdomains (to be referred to as local element system, or just element for simplicity). One of the finite element systems used in the present study is shown in figure 1. Since the size and shape distribution as well as the order of the interpolation function all effect the accuracy and efficiency of the FE solution. An optimum combination exists for obtaining the most efficient computer program for a given accuracy requirement. Various sizes and shapes of elements have been studied.

Once a finite element system is selected, such as the one in Figure 1, stream function Ψ in equation 10 in approximated within a finite element (subdomain) by

$$\Psi(x,y) = \Omega_n(x,y)\,\Psi_n \qquad 13$$

with $n = 1,2...r$ (r = total number of nodes in the element considered) $\Omega_n(x,y)$ are interpolation functions, and Ψ_n are values of Ψ at nodal points of the element so that Ψ_n's are not function of spatial coordinates. Usually, it is more general and convenient to define the interpolation functions in a natural coordinate system. For a typical quadrilateral isoparametric element, the 'linear' interpolation functions are of the form:

$$\Omega_n = \tfrac{1}{4}(1+\xi_n\xi)\,(1+\eta_n\eta). \qquad 14$$

The next step is to establish a functional of the boundary value problem based on one of the several concepts, such as Variational Method after Rayleigh-Ritz, Weighted Residual Method, etc. An approximate solution of the problem is obtianed by extremizing this functional. Two methods, namely, the Rayleigh Ritz's and the Galerkin's, have been found very suitable for the FE Analysis of fluid flow problems. Although both of these two methods have been applied in the present study, the FE Solution of cascade potential flow based on the Galerkin's Method is presented below for illustration purpose.

To obtain the FE Equation of equation 10, the Galerkin's integral (after applying the Green-Gauss theorem) becomes

$$\int \Psi_{,i}\, n_i\, \Omega_n\, dA - \int_V \Psi_{,i}\, \Omega_{n,i}\, dv = 0. \qquad 15$$

Substituting the interpolation function of Ψ, equation 13, into the above equation, one obtains

$$\left(\int_V \Omega_{n,i}\Omega_{m,i}\, dv\right) \Psi_m = \int_A \Psi_{,i}\cdot n_i\, \Omega_n\, dA \qquad 16$$

This equation may be simplified as

$$A_{nm}\,\Psi_m = F_n \qquad 17$$

where: $A_{nm} = \int_V \Omega_{n,i}\,\Omega_{m,i}\, dV$

$F_n = \int_A \Psi_{,i}\, n_i\, \Omega_n\, dA$

It should be noted that the summation convention is used for all repeated indices.

Usually it is more convenient to evaluate the coefficients, A_{nm} and F_n, in the local natural coordinate system, especially for the isoparametric formulation. Because, in many cases, the results of the intergration are independent of the element, i.e. they are the same for all elements. One should note that if the integration is performed in the local natural coordinate system the Jacobian of the coordinate transformation is involved. The integration may be carried out analytically by hand or computer using symbolic computing languages, or numerically using Gaussian quadrature.

Having evaluated the coefficients of A_{nm} and F_n for every element, one can assemble all the element equations into a set of global equations in the following form:

$$A_{ij}\,\Psi_i = F_i \qquad 18$$

Probably the most important step of the solution of a boundary value problem is to impose the proper Boundary Conditions. For the case of Laplace Equation in terms of stream function the boundary conditions needed are of the Dirichlet type, i.e. the value of the stream function is specified constant along a solid boundary, because of the appearance of "natural boundary conditions" in variational methods. The periodicty of flow properties in the cascade flow problem in terms of stream function can be taken into account simply by specifying the upper and lower boundaries with different constants. The difference between these two constants should depend on the circulation based on the inlet and outlet conditions.

In general, any type of boundary conditions may be represented by

$$\Psi_i = q_{it}\,\overset{*}{\Psi}_t \qquad 19$$

with t representing only the interior nodes. Combining equation 19 with the global equation 18, one obtains

$$\overset{*}{A}_{st}\,\overset{*}{\Psi}_t = \overset{*}{f}_s \qquad 20$$

where: $[\overset{*}{A}] = [q]^T\,[A]\,[q]$

$[\overset{*}{f}] = [q]^T\,[f]$

This is usually referred to as the Modified Boundary Matrix Method. One should note that the resulting equation 20 is much smaller than that obtained from the Langrange Multipliers Method.

The next step is the solution of the final set of global FE equations with boundary conditions imposed, eg. equation 20.

Many existing scientific subroutines based on Gauss Elimination, Matrix Inversion, Newton-Raphson Method, and Method of Steepest Descent have been found effective. After the nodal values of the function having been computed, one may, if needed, obtain the value of the function at any point in the flow field using the interpolation function. Other flow properties such as velocity and pressure may also be calculated from the results of stream function or velocity potential solution.

For the case of a time dependent problem, the basic approach described so far is exactly the same. The only additional work required is the discretization of the time derivative, which can be easily done using the finite difference method or the Taylor series expansion. The resulting equations are then solved by a time-marching scheme.

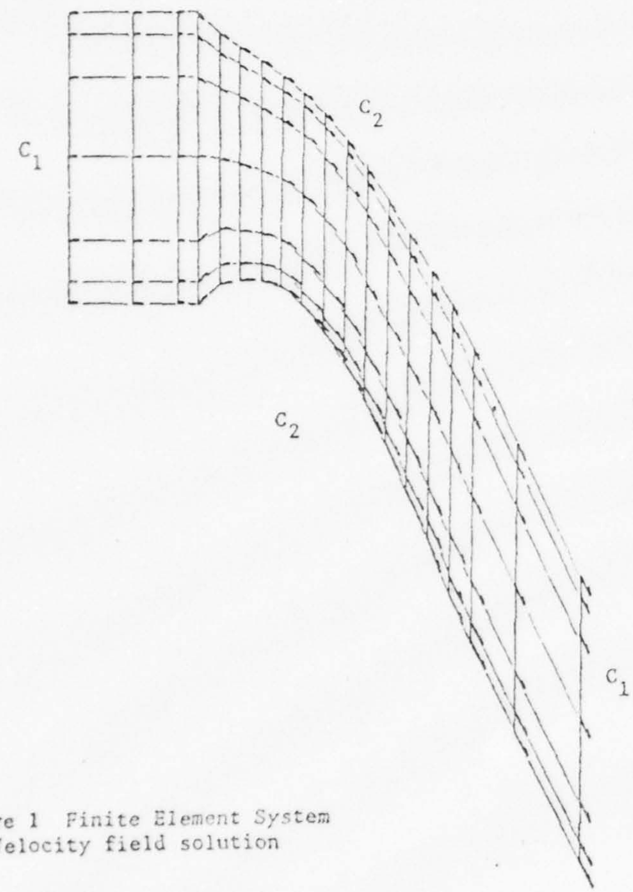

Figure 1 Finite Element System and Velocity field solution

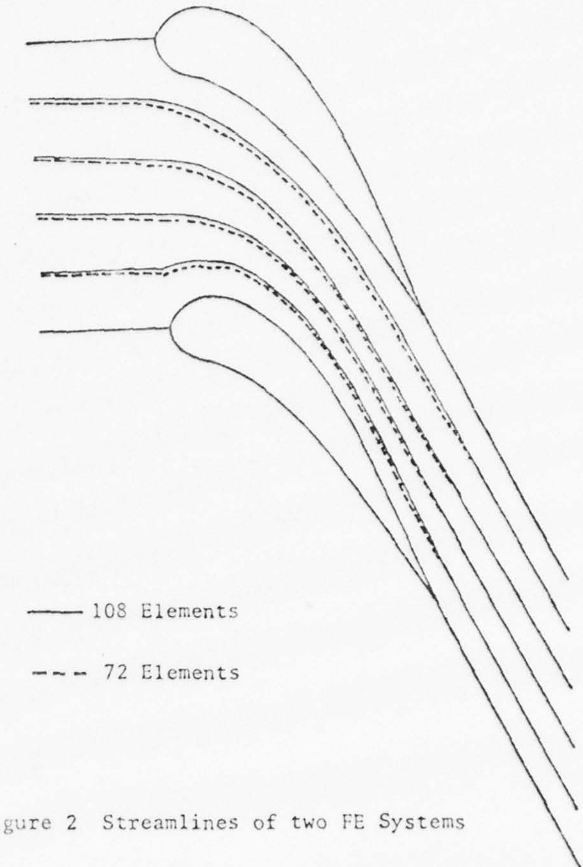

Figure 2 Streamlines of two FE Systems

CONCLUSION

A computer scheme for simulating the 2-d cascade flows based on the finite element method has been developed with computer codes in Symbolic Languages for handling the model formulation, simplification and discretization and computer codes in FORTRAN IV for carrying out the numerical solutions.
The results (see Figure 1 & 2) of testing cases of potential flows confirm that the model is fairly accurate. Better results have been obtained using element system with more (smaller) elements at the cost of computing time, which is found being proportional to the cubic power of the number of elements used. The pressure distributions along the blade surfaces are in good agreement with both experimental and analytic solutions published.

Although it is indeed true that the application of FEM in solving turbomachinery flows is still in its infancy; the advantages of its generality and simplicity in both mathematical formulation and numerical solution, its capability of accomodating complex boundary geometry, and its flexibility in adjusting the size/shape distribution to achieve an optimum accuracy and cost requirement have all experienced by the authors of the paper. Therefore, it is highly possible that upon further improvements the FEM will be the method to simulate realistic 3-d turbomachinery flows.

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REFERENCES

Bosman, C., El-Shaarawi, M.A.I. (1976) Quasi-3-D Numerical Solution of Flow in Turbomachines. ASME 76-FE-23.

Davis, W.R., Millar, D.A.J. (1972) A Matrix Method Applied to the Analysis of the Flow Past Turbomachine Blades. Calton Univ. Rept. ME/A 72-7.

Gelder, D. (1971) Solution of the Compressible Flow Equations Int. J. Num. Methods in Engineering, 3, 35-43.

Hirsch, Ch., Warzee, G. (1976) A Finite Element Method for Through-Flow Calculations in Turbomachines. ASME 76-FE-12.

Japiske, D. (1976) Review-Progress in Numerical Turbomachinery Analysis. J. Fluids Engr. 98, Series I, No. 4, ASME Trans.

Katsanis, T. (1968) Computer Program for Calculating Velocities and Streamlines on a *Blade-to-Blade Stream* Surface of Turbomachine. NASA TN-D-2809.

Kurzrock, J.W., Novick, A.S. (1973) Transonic Flow Around Compressor Blade Elements. Vol. I & II, AFAPL-TR-73-69.

Mach, K.D. (1976) An Alternating-Direction-Implicity Solution of Subsonic Cascade Flow. USAF Aero Prop. Lab. AFAPL-TR-75-80.

Norrie, D.H., DeVries, G. (1971) The Application of the Finite Element Technique to Potential Flow Problems. ASME 71-APM-22.

Senoo, Y., Nakase, Y.(1971) A Blade Theory of an Impeller with an Arbitrary Surface of Revolution. ASME 71-GT-17.

Thompson, D.S. (1974) Flow Through a Cascade of Airfoils in Finite Element Methods in Flow Problems, 707-720, UHA Press.

Wang, S.Y. (1975) A Study on Numerical Methods for Computing Transonic Flows in Turbomachines. USAF-ASEE Summer Program Res. Rept.

White, J.W., Kline, S.J., (1975) A Calculation Method for Incompressible Axisymmetric Flows. Standord Rept. MD-35.

THERMODYNAMIC-MECHANICAL MODELLING OF THE TURBOCHARGED DIESEL ENGINE

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INTRODUCTION

The numerical modelling of the thermodynamics of internal combustion engines, which are used to power automobiles, lorries, trains, planes and ships, and to drive electric generators, pumps, compressors and many other machines, is a widely employed method to develop these engines further. It allows one to predict the effects of changes in operating conditions, ambient conditions and design without costly experiments. With the advent of electronic computers, initial models were concerned with advancing the cycle computations which have a long history going back to the Carnot, Stirling, Otto and Diesel cycles. More recently these models have been expanded to include components and processes outside the engine cylinder.

One such component is the exhaust turbocharger. Its function is to increase the amount of air trapped in the cylinder so that more fuel can be burnt and a greater power output obtained. To this end, the density of air is increased in a compressor before it enters the engine; the mechanical energy absorbed by the compressor is obtained from a turbine driven by the exhaust gases. Turbocharged diesel engines (as opposed to naturally aspirated ones) are now used exclusively in ships, locomotives and power stations, and increasingly in heavy vehicles.

Over the years, the boost provided by turbocharging has gone up from a fraction of, to several times, atmospheric pressure, and the turbocharger has changed from an auxiliary to a major engine component generating and absorbing power up to half of that of the engine, with a consequent increase in the need to accurate modelling. None of this power becomes directly available at the engine shaft; it circulates within the system, and the exhaust turbocharger is said to be free-floating and self-governing. This causes one of the main difficulties in modelling the interaction between the engine proper and the

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NOMENCLATURE

C_v	constant volumn specific heat
E	internal energy
F_i	component of body force in X_i-direction
P	pressure
R	gas constant
T	temperature
t	time
u	velocity component in X_1 (or X)-direction
v	velocity component in X_2 (or Y)-direction
X,Y	global and physical coordinates

Greek

δ_{ij}	Kronecker delta
K	thermal conductivity
λ	bulk viscosity
μ	dynamic viscosity
ξ,η	local coordinates
ρ	density
ϕ	velocity potential
ψ	stream function
Ω_n	interpolation function

CHAPTER I

INTRODUCTION

A better understanding of the fundamental characteristics of the flow through turbomachines has become increasingly important to both experimental researchers and turbomachinery designers. It offers the experimentalist a wiser choice of test parameters, and provides the designer with the preliminary design information.

As the performance as well as the efficiency requirements have become more and more demanding the turbomachines have to be operated at much heavier aerodynamic loads and much higher inlet speed or temperature. The real flow phenomenon within the turbomachines has become extremely complex. It is a truly three-dimensional, unsteady, non-uniform, rotational and turbulent flow of viscous, compressible, and heat-conducting fluids involving boundary layer separation, cavitation and reattachment effects, as well as the shock wave interactions. Besides, the complicated boundary geometry causes additional analytical difficulty. The mathematical model of real flow through turbomachines includes a set of non-linear partial differential and algebric equations (to be described in Chapter II). Recently, the classical, over-simplified analytic solutions which provided useful information for low-speed turbomachines in the past have been found inadequate for modern turbomachine design. The empirical approach also used in the past as a primary tool in the development of turbomachines has been found too costly as well as time-consuming due to the fact that there are too many parameters having to be considered before an optimum configuration of the high performance turbomachine can be determined. Futhermore, a failure occuring during the development of a new turbomachine not only requires millions of dollars to redesign, manufacture, and test its replacement, but also seriously impedes the development cycle. Consequently, the numerical approaches or computer simulations of more realistic turbomachinery flows have become more and more important for preliminary design information and for guiding the intelligent experimental studies to reduce unnecessary tests.

Steady progress in the field of turbomachinery numerical calculations have been achieved during the past twenty years or so. The methods of numerical approaches have changed significantly from one-dimensional passage calculations to two-dimensional transonic calculations as well as the three-dimensional inviscid and boundary layer problems. According to the fundamental work of Wu[1], the three-dimensional inviscid flow equations are formulated into two sets of two-dimensional inviscid flow equations on pseudo-orthogonal surfaces. One of these flows is located in blade-to-blade surfaces (the S1 surfaces); the other flow lying on hub-to-shroud surfaces (the S2 surfaces). Information obtained from solutions in blade-to-blade planes (S1) must be used in the meridional solution (S2) and vice versa in an iternative solution scheme. Approaches are made today with either streamline curvature, finite difference or finite element formulations for the solution of realistic turbomachinery flow problems with some success. The streamline curvature method has been widely used in the solution of turbomachinery flows. The finite difference approach is being developed and the finite element method applied to turbomachinery flows is still in the stage of infancy.

Since the present work deals with two-dimensional flow through a cascade of blades; so only a few contributions closely related to blade-to-blade flows will be cited in the following. The streamline curvature method and the finite difference method are now widely used in the solution of the blade-to-blade equations for steady compressible flow. In the first method, the streamline curvature method, a differential equation for the gradient of steamwise velocity along the normal or near normal to the streamline is written in terms of the assumed radius of curvature of the streamline. This equation is integrated across the blade passage to give the velocity. This method appears to give satisfactory answers for isentropic transonic flow, but its validity in flow with shocks must be open to doubt. In the second method, the finite differnece method, the equation is written in terms of a stream function satisfying the continuity equation, and solved by finite difference schemes of matrix inversion or relaxation. Recent developments are in the category of the explicit finite difference solution of the time-dependent Navier Stokes equation in quasi-conservative form governing the transonic flow of viscous and compressible fluid. Wang and Mach[2,3] had studied this approach carefully and attempted to improve its computation efficiency as well as its accuracy. Unfortunately, it has been found that not only its efficiency may not be improved significantly, but also its many simplifying assumptions are unsound, such as, it assumes that the location of stream surfaces is known a priori without successive corrections, the flowfield variations normal to the stream surfaces are ignored, the relation between the inlet and outlet stream surface locations is linear, the fluid is perfect gas with constant viscosity and heat-conductivity, etc. The improvement of this model requires the redevelopment of the governing differential equations involving several complicated coordinate transformations. Furthermore, it is anticipated that the computation time of the improved model will be increased substantially. On the other hand, the results of this method are in good agreement only with the exact incompressible solution, except near the leading edge where grid accuracy is frequently compromised. Therefore, it may be wise to start the development of an entirely new approach.

A promising solution technique should be simple, general, efficient, and capable of handling models of higher levels of sophistication through a systematic process of improvements. The finite element method is chosen not only because it has these advantages; but also because it is extremely suitable for the complex geometry of turbomachinery flows and capable of simulating three-dimensional flows directly requiring only minor modificaiton of the two-dimensional solution procedure. The finite element method is not new, but considered well-established in the turbomachinery world as a tool of the structural designer. Using this tool, many previously intractable problems in structural design and vibrational analysis have been reduced to routine calculations. In comparison with these spectacular successes, exploitation and application of this tool by the aerodynamic designer has lagged behind.

Recently, the effort in the application of finite element methods to problems of fluid flow has been increased drastically. Numerous reports have been published. Martin[4], Argyris, Maryczek, and Scharph[5] were among the earlier ones to study the application of finite element methods to fluid flow problems. The potential flows were treated by Norrie and De Vries[6]. Leonard[7] and Gelder[8] studied the linearized compressible flow problems by finite element methods. Olson[9] and Baker[10] have developed some finite element method algorithms for viscous incompressible flow, primarily for environmental studies. The unsteady incompressible flow around an oscillating body was investigated by Bratanow, Ecer, and Kobiske[11]. Chan and Brashears[12] applied the finite element to the analysis of time-dependent transonic flow around a symmetric airfoil executing harmonic motion. Some initial results of Thompson's[13] in the application of the finite element method to the flow through a cascade of airfoils have been encouraging. Even though the case he studied was only for an inviscid and incompressible ideal fluid, the extension to the cases of compressible, viscous fluids is very promising. More recently, Hirsch and Warzee[14] obtained the solution of the meridional through-flow in an axial-flow machine using the finite element method, and the solution of transonic flow over an airfoil based on the finite element method was reported by Chung and Hooks[15]. They also proved that not only the results are satisfactory, but also the method is simpler and more versatile than other methods existing today.

The present work is concerned with steady, inviscid, incompressible, two-dimensional flow through a cascade of blades. And, the main objective is primarily the development of computer codes for model formulation and solution using the finite element method. All codes developed here are as general as possible, so that they may be used to solve more sophisticated models in the near future with as little modification as possible. The attractiveness of the finite element method for computing the flow through turbomachines is its capability of modeling three-dimensional viscous and compressible flows. A detailed description of the mathematical formulation and the solution of the method are presented in the following chapters. The results are in good agreement with experimental and analytic data in the open literature.

CHAPTER II

MATHEMATICAL MODEL FORMULATION

Although the case of this thesis is the simplest case, two-dimensional, inviscid, and incompressible flow; the simulation of the realistic, three-dimensional, transonic flow of viscous, compressible, and heat-conducting fluids is our overall objective. Therefore, the model formulated is intended to be general and feasible for modification to treat models of higher levels of sophistication in the future. For three-dimensional turbomachinery flow of a viscous, compressible and heat-conducting fluid, the generalized governing differential equations in tensor notation are:

continuity equation

$$\rho_{,t} + (\rho V_i)_{,i} = 0 \tag{1}$$

momentum equation

$$V_{i,t} + V_j V_{i,j} = P_{,i}/\rho + F_i + \tau_{ij,j} \tag{2}$$

energy equation

$$\rho(E_{,t} + V_j E_{,j}) = -\rho V_{i,i} + (\kappa T_{,i})_{,i} + \tau_{ij} V_{i,j} \tag{3}$$

equation of state

$$P = \rho R T \tag{4}$$

$$E = \int C_v \, dT \tag{5}$$

where

$$\tau_{ij} = (\lambda - \tfrac{2}{3}\mu) V_{\ell,\ell} \delta_{ij} + \mu V_{i,j}$$

and definitions of notations are given in the nomenclature.

The above set of five equations may be solved for the five unknowns Vi, P, ρ , T and E. For a more general case, the state equations determining the functions Cv, λ , K, and μ of the thermodynamic properties should be added.

In order to simplify the governing differential equations, the assumptions made are time-independency, two-dimensionality, constant density, zero viscosity and zero body force. The simplified governing equations become:

continuity equation:

$$V_{i,i} = 0 \qquad (6)$$

The momentum equation is satisfied, if the stream function is assumed to satisfy the irrotationality condition. If the stream function, ψ , or velocity potential, ϕ , formulation is used, the following relations are defined:

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$
$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad (7)$$

By a simple substitution, the governing equation in terms of either the stream function (ψ) or the velocity potential (ϕ) may be written in the form of LaPlace's equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad (8)$$

where the first equation is resulted from the continuity equation and the second equation was derived from the irrotationality condition. In tensor notation, they are:

$$\phi_{,ii} = 0$$
$$\psi_{,ii} = 0 \qquad (9)$$

In general, the choice between velocity potential and stream function *in the finite* element formulation depends on boundary conditions, whichever is easier to specify, as well as on the accuracy of the solution. Although the stream function formulation is presented in detail to describe the solution procedure, the potential flow formulation has been studeid as well.

To impose the proper boundary conditions is very important during the procedures of solution. The stream function gives rise to Dirichlet type boundary condition, i.e., the value of the stream function is specified constant along a solid boundary. For the periodicity of flow properties in the cascade flow problem, the stream function value can be taken into account simply by specifying the upper and lower boundaries with different constants, and the difference between these two constants should be the same for all passages. The velocity potential formulation, however leads to Neumann type boundary conditions, i.e., the derivative of the potential normal to a boundary must be specified. Since the normal component of the derivative of ϕ is zero along a solid surface boundary. The solution of the velocity potential using the Finite Element Method satisfies these boundary conditions (so called natural boundary conditions) automatically. At the entrance and exit of the flowfield, the values of the velocity potential are specified, the gradients of the stream function are not necessary, however, because they are natural boundary condition. One may refer to Thompson's work[13] for details.

CHAPTER III

FINITE ELEMENT SOLUTION

As mentioned previously, the finite element method is chosen not only because it is simple, general, efficient, and capable of handling models of higher levels of sophistication; but also because its suitability for highly irregular boundaries of turbomachinery flows as well as being capable of simulating three-dimensional solution procedure. The finite element method is a systematic procedure through which a continuous funciton is approximated by a discrete model consisting of a set of values of the given function at a finite number of points in its domain together with piecewise approximations of the function over a finite number of sub-domains, called "finite elements". The local approximation of the function over each finite element is uniquely defined in terms of the discrete values of the function at the finite number of preselected points (nodal points) in its domain.

The general approach of finite element solution is outlined below as applied to the Laplace euqation of the present study. First of all, the entire flowfield (to be referred to as the global system) is divided into a set of finite element subdomains (to be referred to as the local element system). There is a wide variety of finite element types from which to choose. They vary in shape, in the order of polynomial used for interpolating functions are developed. For a two-dimensional element, the accuracy in the form of element norm is determined by:

$$\| D^m \psi(x_i) - D^m \hat{\psi}(x_i) \| \leq C \frac{h^{p+1}}{\rho^m} M_{p+1} \tag{10}$$

$$M_{p+1} = \| D^{p+1} \psi \|$$

where: ψ and $\hat{\psi}$ respectively are exact and approximate finite element solutions of the problem respectively,

D^m is mth order partial differential operator,

$\|A\|$ means the energy norm of A,

C is a positive constant,

h is the longest dimension,

ρ is the largest diameter of a circle inscribed in the element considered (see figure 10),

m is the order of the differential equation,

p is the degree of the approximating polynomial or the interpolation function of an element.

We see clearly that both the size (h) and the shape (ρ) are important to the accuracy of the approximate solution. Besides, the local gradient of the solution has to be considered in the final selection of the finite element mesh system in order to obtain accurate results throughout the entire domain. It is also obvious that more accurate solutions may be obtained by using higher degree approximating polynominals within each element without changing the element mesh system. The use of a high order polynomial is thought to improve accuracy sufficiently to sacrifice the increased programming and computing effort. Therefore, an optimum condition of the degree of interpolation functions and the finite element shape/size arrangement may exist to render the best possible computing efficiency. Four different finite systems of various sizes, which are shown in figures 1-4, have been studied for investigating the effect of size.

Once a finite element system is selected, take figure 1 for example, the stream function ψ in equation (9) is approximate within a finite element by

$$\psi(x,y) = \Omega_n(x,y)\psi_n \tag{11}$$

with n=1,2,..., r (r=total number of nodes in the element considered), $\Omega_n(x,y)$ are *interpolation functions*, ψ_n *are values of* ψ *at nodal points* of the element so that the ψ_n's are not functions of spatial coordinates of the element. In the present study, the quadrilateral isoparametric elements are used to approximate the solution over an element. The element mesh system is chosen according to the anticipated flowfield property variation as well as the geometry of the boundary. Therefore, the stream function ψ within a finite element can be written by

$$\psi(x,y) = \Omega_1(x,y)\psi_1 + \Omega_2(x,y)\psi_2 + \Omega_3(x,y)\psi_3 + \Omega_4(x,y)\psi_4 \tag{12}$$

Further details concerning the interpolation function are presented in appendix B.

The next step is to establish a functional of the boundary value problem based on the variational method after the Rayleigh-Ritz or Weighted Residuals Method. An approximate solution of the problem is obtained by extremizing this functional. These two methods, namely, the Rayleigh-Ritz's and the Galerkin's, have been found very suitable for the finite element analysis of fluid flow problems. Here, the finite element equation of the two-dimensional cascade potential flow based on Galerkin's method is derived

$$A_{nm}\psi_m = F_n \tag{13}$$

The details of the derivation using Galerkin's Method is presented in the appendix A. It should be noted that the summation convention is used for all repeated indices. Equation (13) is called a local element equation defined in each finite element subdomain where n & m are element nodal numbers. The coefficients A_{nm}, and F_n are evaluated, in the local coordinate system, using the isoparametric formulation. One should note that since the integration is performed in the local coordinate system, the Jacobian of the coordinate transformation is involved when the local element equations are assembled into the global system in the original physical domain. The integration may be carried out analytically or numerically. The computer program for evaluating these integrals using Gaussian quadrature has been developed. Experience has shown that four point Gaussian quadrature yields sufficiently accurate results. The computer program using the MATHLAB (a Symbolic Computing Language) for this purpose has also been developed.

Having evaluated the coefficients of A_{nm} and F_n for all elements, we can assemble all the element equations into a set of global equations in the following form:

$$A_{ij}\psi_j = F_i \tag{14}$$

where:

$$A_{ij} = \sum_{e=1}^{E} \Delta_i^n A_{nm}^e \Delta_j^m$$

$$F_i = \sum_{e=1}^{E} \Delta_i^n F_n^e$$

and the incidence symbol Δ_i^n is defined to be either one or zero depending on whether or not the local element nodal number n coincides with the global nodal number i. Here, e indicates the element number, E equals the total number of elements, and i and j designate the nodal numbers of the global system to distinguish from n and m, the nodal numbers of the local element system. One should note that the summation is done over all finite elements, however, only elements directly around the global node will have a contribution. Therefore, the coefficient matrix of the global equation is usually a banded matrix. The computer program developed is able to assemble all element equations automatically.

In a boundary value problem, the most important step of the solution procedure is probably the introduction of the proper boundary conditions needed for the case of Laplace's Equation are of the Dirichlet type,i.e., the boundary conditions are specified constant along a solid boundary surface and flow velocities (Neumann type boundary conditions as far as the stream function is concerned) are assumed normal to the entrance and exit of the flowfield (Figure 5). Since the property of the "natural boundary conditions" appears in the variational principle of a boundary value problem, the unspecified Dirichlet type boundary conditions at the entrance and exit will adopt this property and yield the correct values automatically. The periodicity of flow properties in the cascade flow problem can be taken into account simply by specifying the upper and lower boundaries with different constants, and the difference between these two constants representing the mass flux of the passage and is maintained the same for all other passages. On the other hand, in the velocity potential formulation, the boundary conditions are specified only at the entrance and exit, and no boundary condition is required along the solid surfaces where the solution will adopt the concept of natural boundary conditions. In general, the application of variational methods including the finite element method to solve a two dimensional flow with boundary conditions involving non-zero normal velocity in a stream function, a line integral along the boundary is required.

The boundary conditions may be discretized in written in the form of,

$$q_{ri}\psi_i = 0 \qquad (15)$$

with r = 1,2,...,R (R = the total number of boundary conditions); i = 1, 2,..., N (N = the total number of the global generalized coordinates or total number of global nodes). Two of the commonly used methods, Lagrange Multiplier Method and Modified Boundary Matrix Method, have been found useful for imposing the boundary conditions. The Modified Boundary Matrix Method has been found easier to use, and results a final set of global equations of smaller size.

The next step is the solution of the final set of global finite element equations with the boundary condition imposed. Using the subroutine SIMQ, in the scientific subroutine package, values of the stream function at every unknown nodal points have been obtained. One may, if needed, compute the value of the stream function at any point in the flowfield using interpolation functions. The velocity distribution of the flowfield, the pressure coefficient on the surface of blade, and streamline distribution in the field may also be calculated from the results of the stream function. Computer codes for both evaluating and plotting the pressure and velocity vector have been developed.

CHAPTER IV

RESULTS

The flow phenomena studied in this project is a two-dimensional, steady, inviscid and incompressible flow through a cascade of blades. The computer codes for model formulation and solution using the finite element method have been developed. All codes (main, subroutines, and funcitons) are designed in the general form, so that they can be utilized for the simulation of more complex phenomenon in the future. All the computations were performed at the Computer Center of the University of Mississippi using the DEC-10 computer. The flow charts and description of the main program and subroutines are presented in Appendix D.

Using the stream function formulation of potential flow, the Dirichlet type boundary conditions along the solid surface are specified. The periodicity of flow properties in the cascade flow problem in terms of the stream function can be taken into account simply by specifying a common difference in values of stream functions on the upper and lower boundaries of each flow passage. In the case of the Laplace equation in terms of the velocity potential, the values of the velocity potential at the entrance and the exit are specified and values of velocity potentials for the rest boundaries can be automatically evaluated by the FEM, a variational approach.

Applying the program developed to the case of a cascade of ovals in potential flow, produce a result comparable to that of the Thompson's. The values of the stream function at any point in the entire flowfield can be evaluated by this computer program either as direct solutions of the global equations or results of interpolation functions. Thus, streamlines in the entire flowfield can easily be plotted (Figure 5) with a CALCOMP plotter. The pressure distribution on the surface of blades is shown in figure 7. As expected, the speed on the front half of the pressure surface of the blade is nearly a constant, and accelerating for the rear part of the blade. In figure 10, the magnitude and direction of the velocity vector at every nodal point in the field are plotted. The velocity gradients in the blade-to-blade direction at the entrance is approximately zero and a certain constant respectively. The results obtained by the finite element solution are in good agreement with experimental, analytic, and other approximate solutions published.

Figure 9 shows that higher accuracy can be obtained by using more elements and/or smaller elements. By testing different parts of the program, it was found that most of the computing time is used during the assembling process of the coefficients of the global equations from the coefficients of the local element equations. In order to store this incidence matrix, it requires a large number of computer memory locations and also takes a lot of computing time to search for the coincidence between the local nodal number and the global nodal number. A plot of the amount of computing time required to solve the entire problem against the number of elements used to approximate the problem is given in figure 8. By examining these curves, it is found that the computing time is clearly proportional to the cubic power of the number of elements used for the case of assembling by incidence matrix. Applying the improved assembling technique (see Appendix C for details), the computing time required for solving a typical case has been reduced by a factor of six.

CHAPTER V

CONCLUSIONS

A finite element model of the steady, two-dimensional, potential, cascade flow has been established. Numerical results for various cases have been generated using a Fortran IV Computer Code/including several subroutines, functions, as well as a main program. Results of typical cases, such as ones plotted in figures 5, 6 and 7, conform well with the potential cascade solutions. By examining the velocity vector plot (figure 9), one can see that the magnitude of the velocity increases where the cross-seciton area of the flow path decreases and vice versa. The direction of the velocity at every node on the pressure and suction surfaces has been calculated and found tangent to the blade surface. It further shows the flowfield obtained from the finite element model is physically sound.

To further verify the correctness of the Finite Element Solution, the normalized static pressure distribution along the pressure and suction surfaces of the cascade flow is reduced from the stream function solution and plotted in Figure 12 to compare with the Delancy's[16] result based on the highly sophisticated finite difference made as well as some experimental data of the same cascade configuration provided in Delancy's dissertation. The deviation between the finite element solutions and Delancy's result is primarily due to the difference in the basic model assumptions. The present model is based on the incompressible fluid assumption; whereas Delancy's model was based on a subsonic compressible flow assumption. The primary reason for comparing with Delancy's results is the fact that his cascade geometry is adopted for the present study. In the same figure, one can also see the fact that the pressure distribution reduced from the velocity potential solution is in better agreement with Delancy's results than that from the stream function solution. It is generally true that the velocity potential solution is more accurate than the stream function solution. However, the stream function formulation is more convenient in specifying the boundary conditions. During the present study, the thin trailing edge of the airfoils if used, so that the Kutta condition at the trailing edges is not needed. It has been observed numerically that a slight increase in trailing edge thickness results a large change in flowfield properties, especially the static pressure distribution. This observation demonstrates the importance of satisfying the Kutta condition at the trailing edges.

It should be emphasized again that the primary objective of the present study is to develop a general computer code based on the finite element method for solving the two-dimensional cascade flows. The potential flow model is chosen to serve as a test case for verification purposes. Based on the aforementioned results, it is obvious that the computer code is satisfactory and can generate reasonable flowfield characteristics from a realtively simple element system. Some effort has been devoted in improving the efficiency of the finite element model. Although an analytic way to determine the optimum configuration (size-shape) of the finite element system for obtaining a reasonably accurate result at least computing cost has not yet been established; the relation between computing time and number of elements has been established based on the computer experimentation of several different finite-element system configurations shown in figure 1-4. It is seen in figure 8 that the computing time is approximately proportional to the cubic power of the total number of elements used to discretize the flowfield. Using this curve one can, at least, estimate how much more he has to pay for the better accuracy he gets by increasing the number (reducing the size) of elements. In the same figure, one also sees that the computing time required to calculate the flowfield using a special assembling technique for obtaining the global equation is considerably reduced. Although this special (or improved) assembling technique is not as general as the incidence matrix scheme; it is definitely more attractive for developing a production program. The basic concept of this special assembling technique is briefly described in Appendix C, and the details of the scheme is in the computer program.

One should also note that almost all subroutines and functions developed from the present study are in a form as general as possible, in order that they can be used in the future to simulate cascade flows of higher level sophistication with little or no modification.

It is generally agreed to the fact that the computing time for simulating a typical boundary value problem with moderately irregular boundary geometry by the finite difference and finite element methods are comparable at the present state-of-the-art. In view of the fact that the finite difference schemes have been in existance much longer and thus have much more refinements built in to them than that of the finite element method, which is still in its infant stage, as far as flowfield analysis is concerned. It is quite clear that the finite element modeling of the cascade flow is more promising in the future, because there is plenty of room for refinement.

CHAPTER VI

RECOMMENDATION

It is quite obvious that the realistic turbomachinery flow is a three-dimensional, unsteady, non-uniform, and turbulent flow of a viscous, compressible, and heat-conducting fluid with additional complications of boundary layer separation, cavitation and even shock wave interactions for some cases. Although the computer program developed from the present study is for the simplest case, the two-dimensional, potential cascade flow; some basic subroutines are directly applicable to more general and sophisticated models as well, and some other subroutines require only minor modifications. With this basic and quite general computer code in hand, one shouldn't have to much difficulty to attack the cascade flows of higher level sophistication.

Due to fact of small flow passage, the viscous effects should be considered. An incompressible and viscous model for the cascade flow may be a logical extention of the present study. The comparison of the proposed model with the inviscid model will provide a better understanding of the viscous effects in the two-dimensional, low speed, cascade flow.

The next model of interest may very well be the three-dimensional, viscous, and incompressible flow in an axial-flow turbomachine. From this investigation, the effects of secondary motions may be explored. Since the finite element models of two-dimensional and three-dimensional cases are not that much different, which is not the case for the finite difference schemes; the study of three-dimensional case should be completed in a short period time.

In addition to the above two models proposed, one can increase the model complexity one step at a time in a systematic manner. Finally, the truly realistic phenomena may, one day, be simulated satisfactorily.

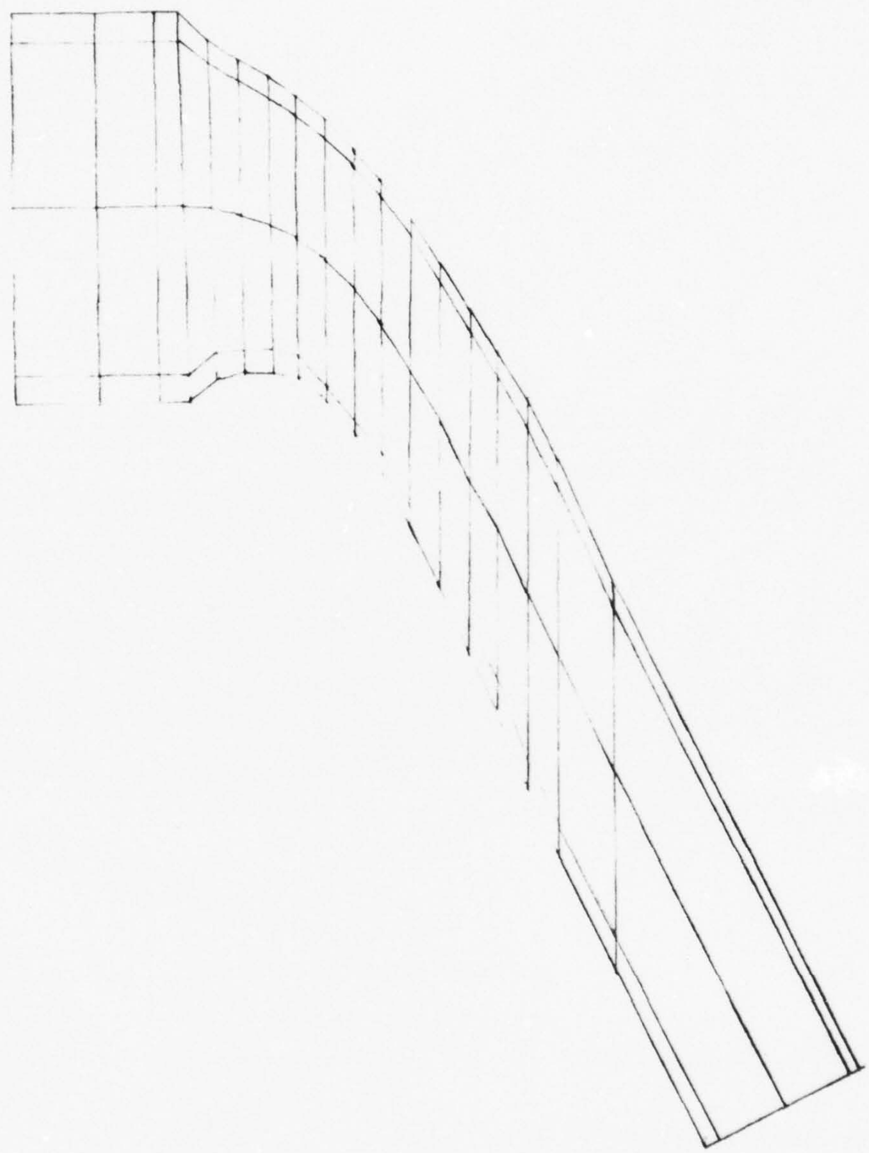

Figure 1. Finite Element System (72 elements)

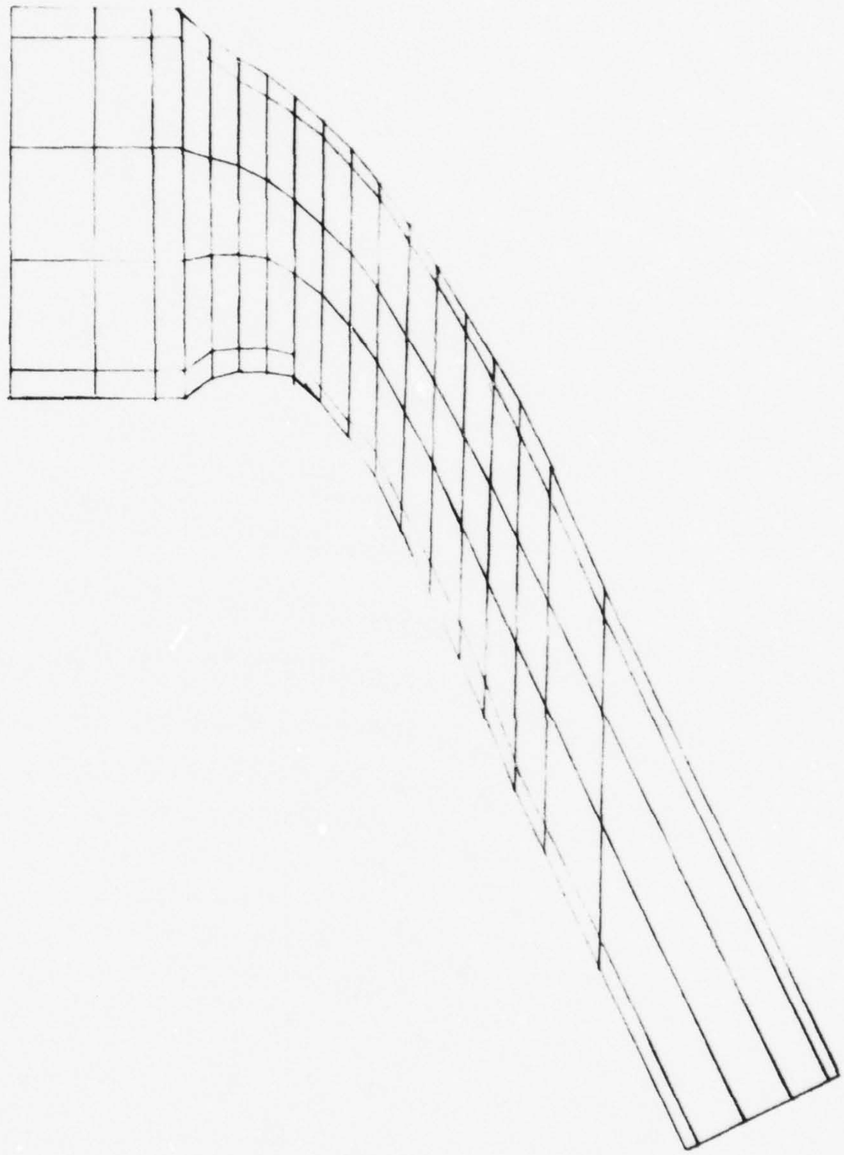

Figure 2. Finite Element System (90 elements)

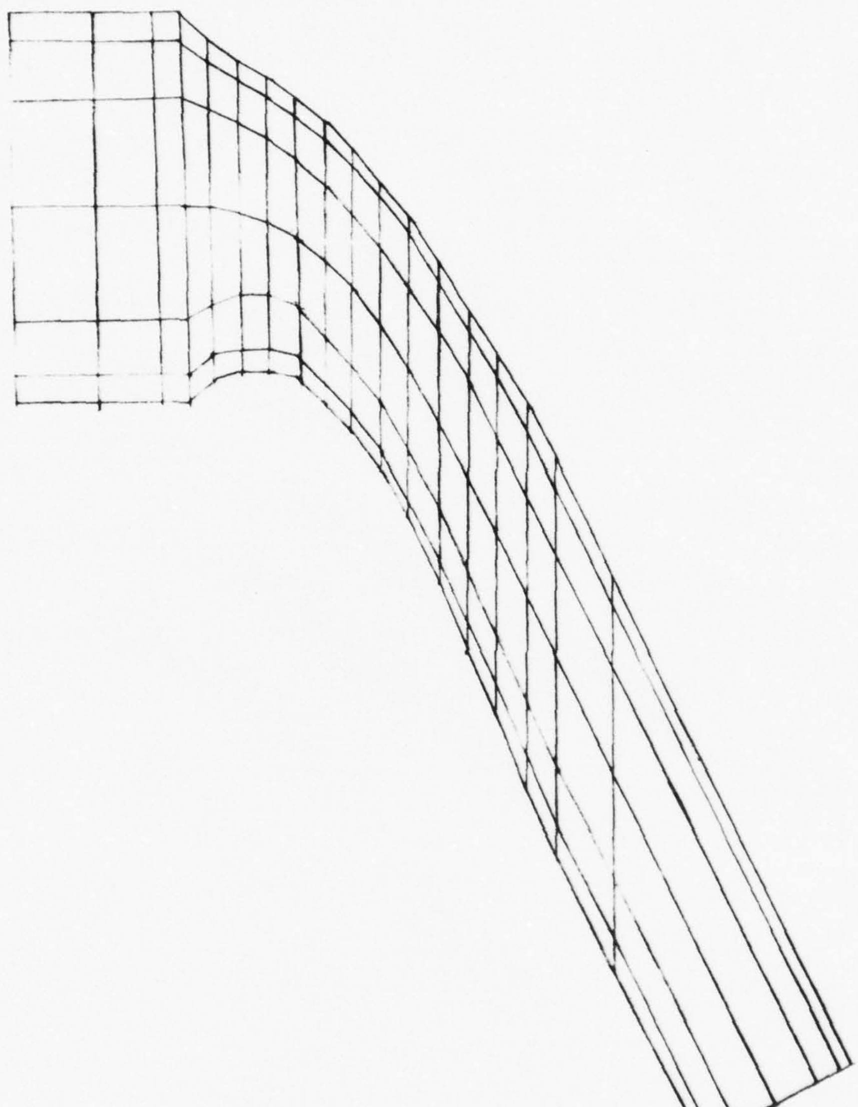

Figure 3. Finite Element System (108 elements)

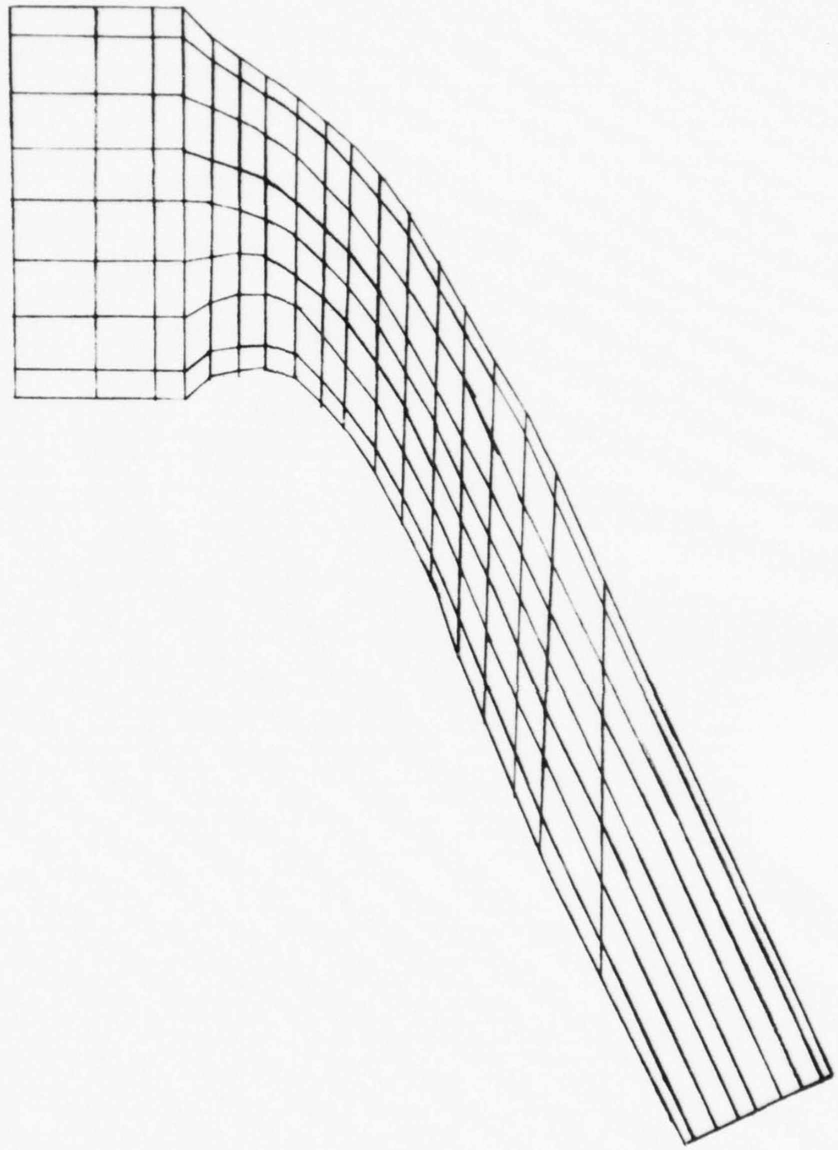

Figure 4. Finite Element System (144 elements)

Figure 5. Boundary Conditions for Stream Function Formulation

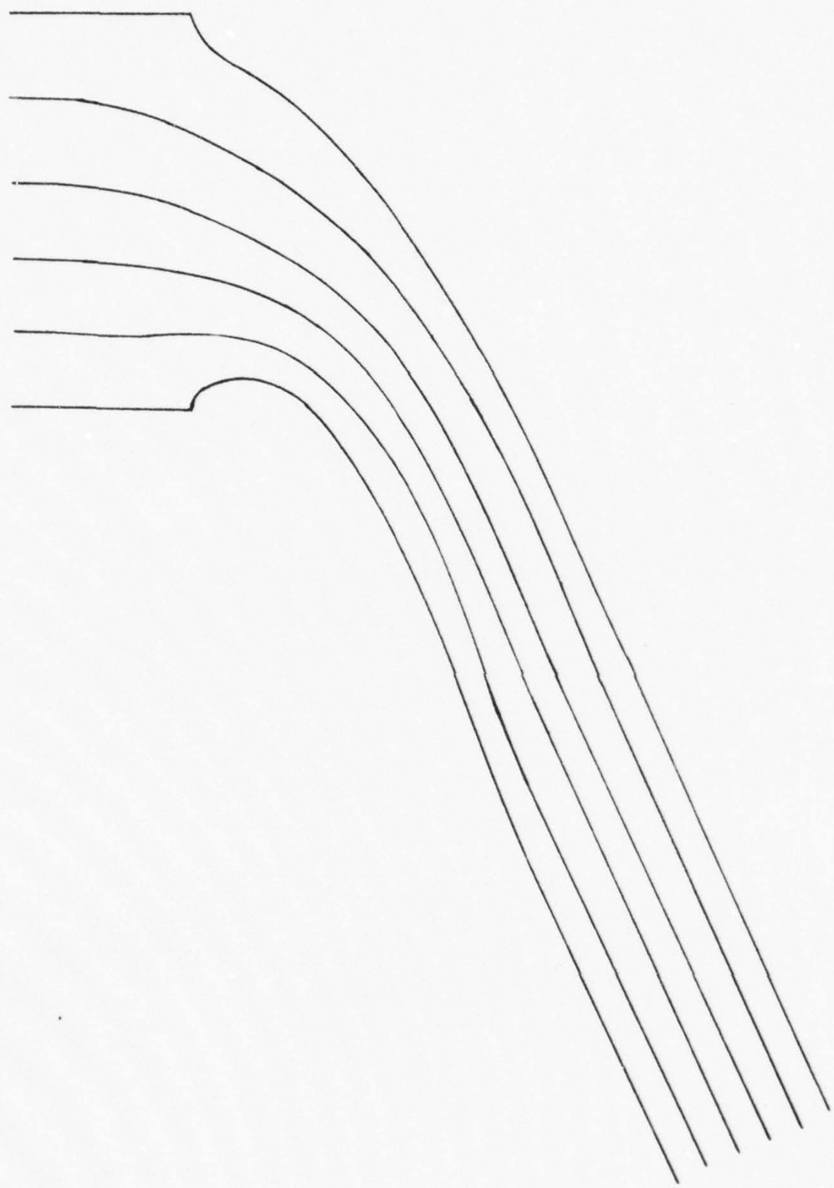

Figure 6. Streamlines of the Entire Flowfield
(144 elements)

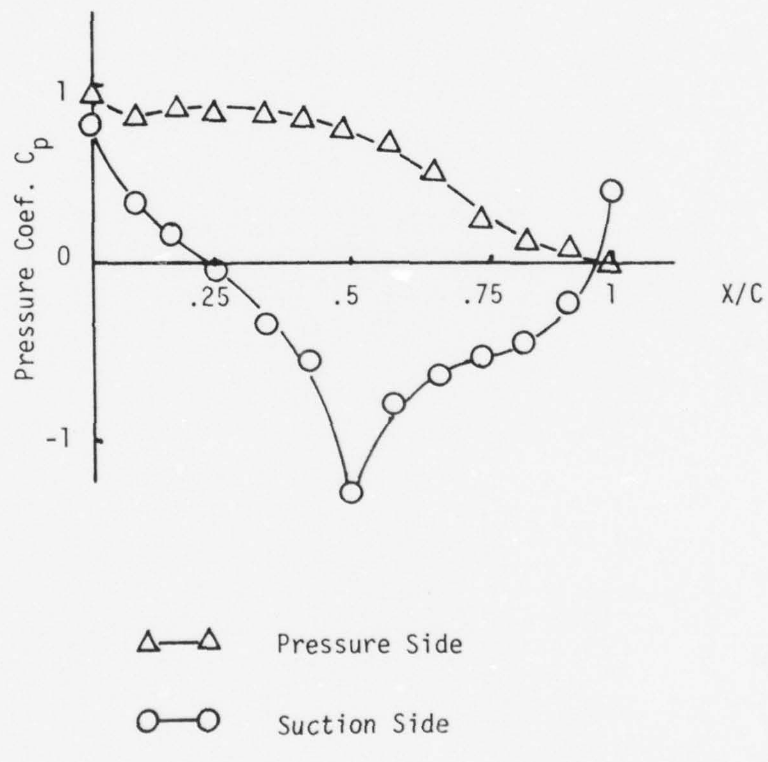

Figure 7. Pressure Distribution on Blade Surfaces

$C_p = (P-P_\infty)/(1/2\rho_\infty V_\infty^2)$

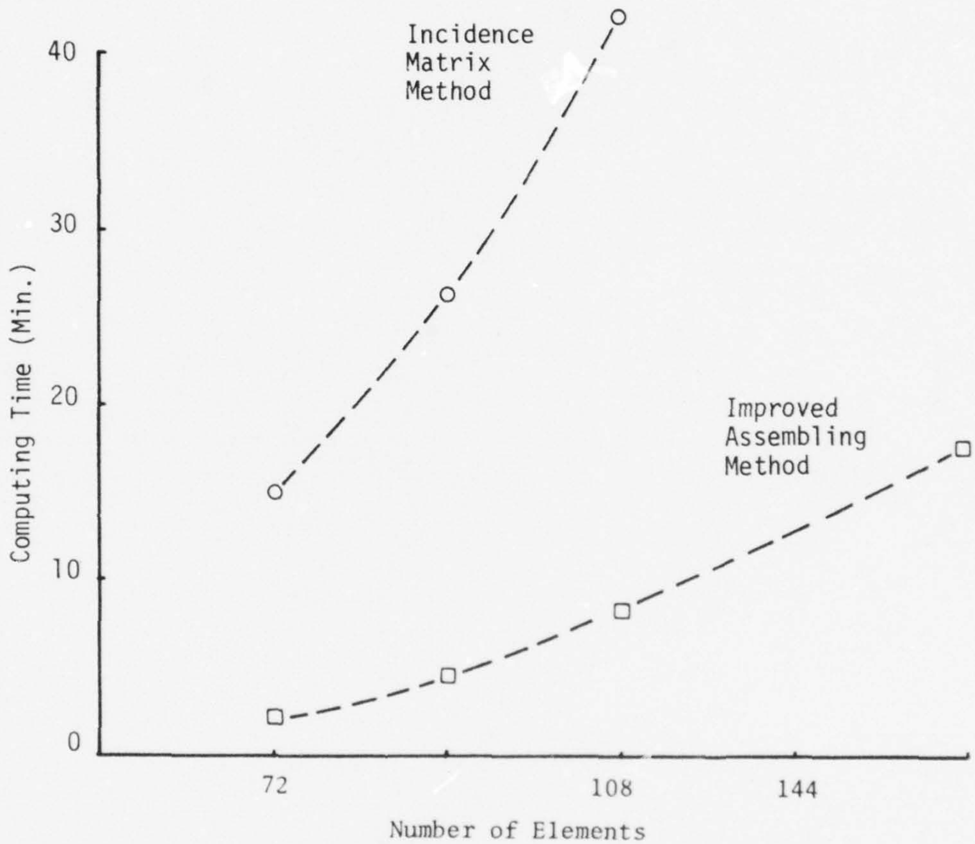

Figure 8. Computing Time Required

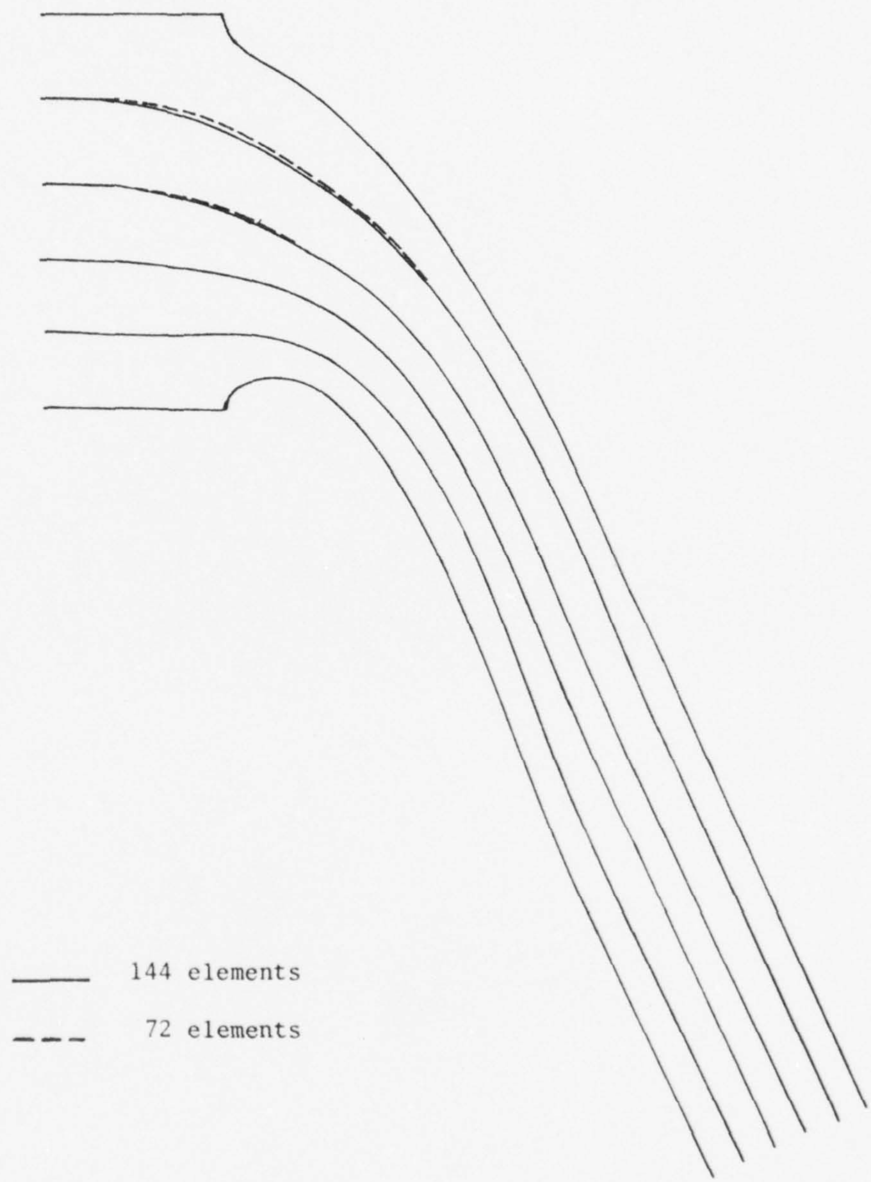

Figure 9. Tendency of Accuracy

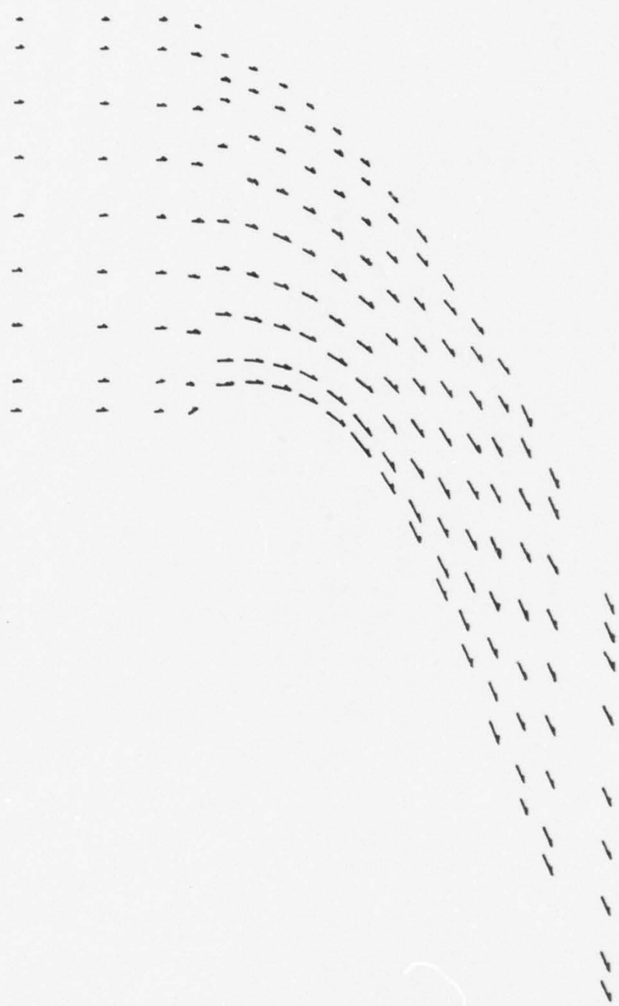

Figure 10. Velocity Vector Plot all Nodal Points

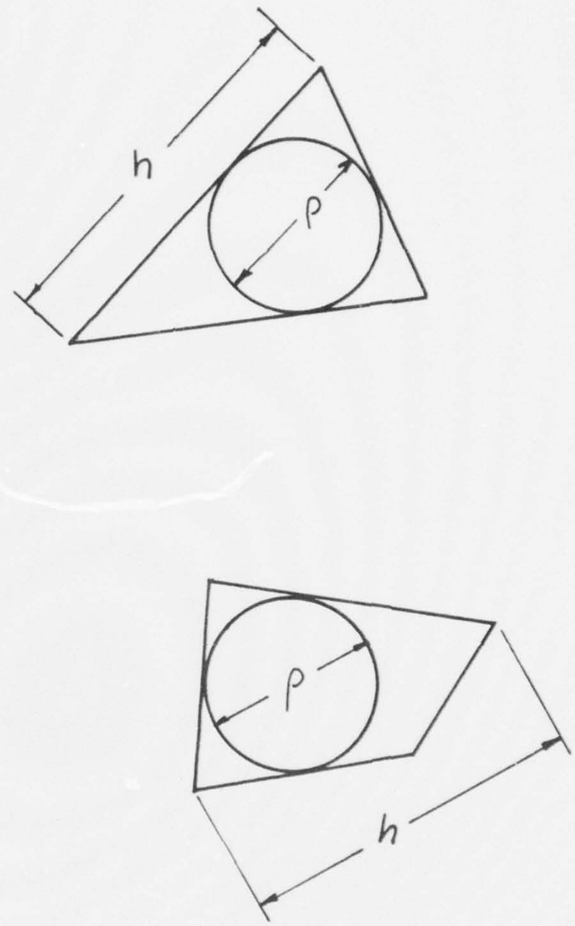

Figure 11. Determination of Size and Shape of an Element h is the longest dimention and ρ is the largest diameter of a circle can be inscribed

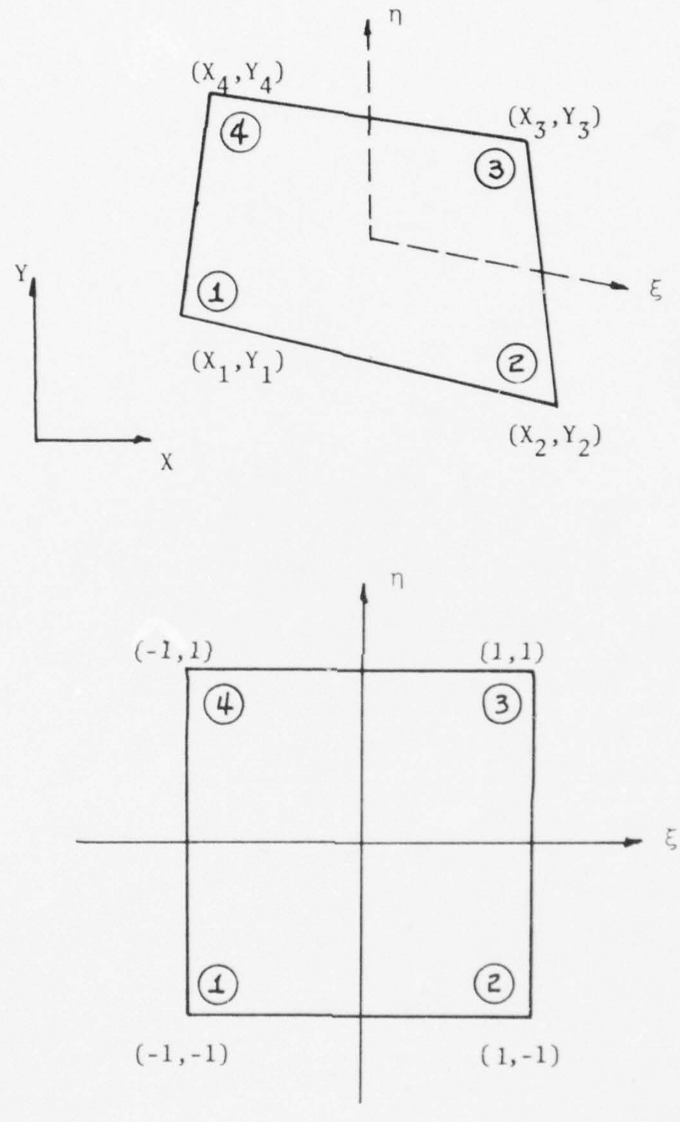

Figure 12. Quadrilateral Isoparametric Element

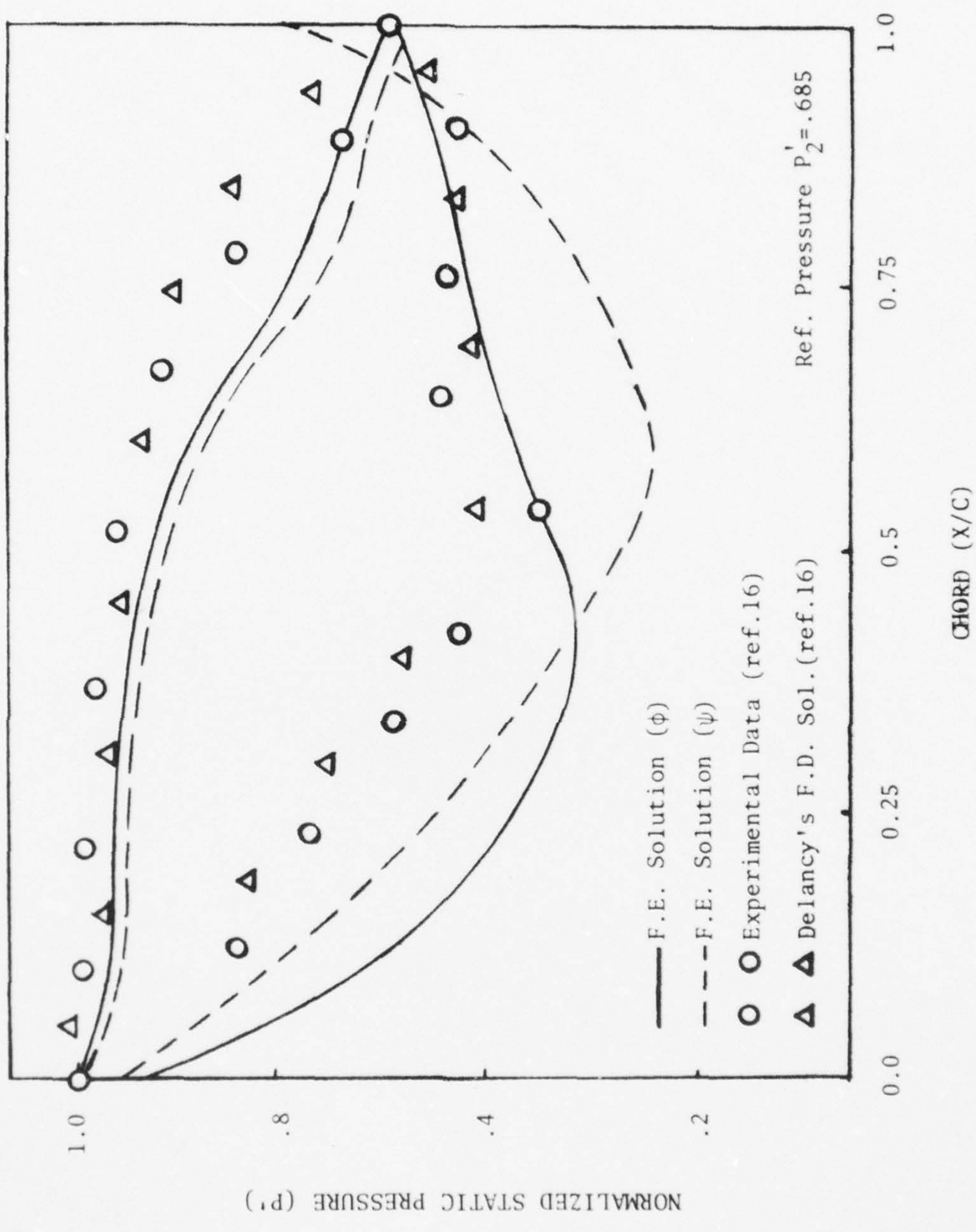

Figure 13. Blade Surface Static Pressure Distribution

TABLE 1

CALCULATED FLOWFIELD PROPERTIES

THE MAGNITUDE OF VELOCITY

0.064	0.060	0.048	0.044	0.065	0.064	0.067	0.071	0.081	0.091	0.104	0.122	0.145	0.159	0.166	0.178	0.189	0.184	0.174
0.066	0.062	0.058	0.061	0.064	0.070	0.074	0.079	0.089	0.102	0.117	0.140	0.155	0.167	0.178	0.186	0.190	0.182	0.175
0.069	0.069	0.070	0.070	0.070	0.078	0.083	0.091	0.101	0.115	0.133	0.154	0.169	0.180	0.188	0.191	0.188	0.181	0.175
0.073	0.075	0.078	0.079	0.082	0.088	0.095	0.104	0.115	0.131	0.151	0.171	0.183	0.192	0.193	0.191	0.184	0.176	0.177
0.078	0.080	0.087	0.091	0.095	0.101	0.109	0.120	0.135	0.151	0.172	0.188	0.194	0.197	0.196	0.190	0.180	0.175	0.178
0.080	0.082	0.092	0.100	0.106	0.116	0.127	0.140	0.158	0.177	0.195	0.203	0.204	0.202	0.196	0.183	0.173	0.174	0.179
0.081	0.082	0.085	0.097	0.121	0.137	0.153	0.169	0.186	0.206	0.212	0.210	0.207	0.201	0.192	0.173	0.170	0.177	0.178
0.081	0.081	0.076	0.052	0.137	0.160	0.182	0.203	0.211	0.244	0.219	0.216	0.209	0.204	0.188	0.156	0.175	0.177	0.176
0.081	0.081	0.072	0.071	0.138	0.159	0.181	0.207	0.215	0.265	0.230	0.227	0.218	0.215	0.193	0.146	0.175	0.177	0.176

THE ANGLE OF VELOCITY

0.00	0.00	0.00	-45.02	-22.79	-30.13	-32.64	-40.72	-45.02	-49.02	-51.37	-56.34	-56.34	-54.30	-61.11	-64.99	-65.04	-65.06	-65.06
-1.16	-5.52	-16.96	-26.25	-25.65	-29.11	-33.80	-39.50	-43.98	-47.79	-50.94	-53.94	-54.12	-57.32	-61.84	-64.33	-65.17	-65.14	-65.14
-3.07	-10.03	-15.76	-16.58	-25.48	-27.93	-33.35	-37.70	-41.17	-45.73	-46.64	-51.58	-54.76	-59.12	-62.75	-64.60	-65.45	-65.23	-65.20
-3.12	-9.55	-14.15	-15.38	-24.76	-26.70	-30.99	-35.01	-39.41	-43.48	-47.50	-51.50	-56.40	-60.74	-63.73	-65.46	-66.11	-65.29	-65.30
-2.47	-8.24	-12.02	-15.23	-16.20	-23.72	-27.56	-33.06	-36.94	-42.08	-47.39	-53.61	-58.39	-62.72	-65.18	-66.52	-66.25	-65.28	-65.28
-1.43	-4.53	-7.51	-9.03	-15.55	-19.07	-23.64	-30.33	-35.28	-42.32	-49.35	-56.51	-61.11	-64.61	-66.74	-67.53	-66.21	-65.20	-65.20
-0.47	1.40	1.35	-6.02	-9.03	-14.18	-20.30	-28.05	-35.31	-44.61	-53.03	-59.06	-63.44	-66.21	-68.14	-67.76	-65.80	-65.08	-65.08
0.00	3.18	19.16	-25.51	1.28	-8.06	-17.53	-27.63	-37.27	-49.41	-56.94	-61.67	-65.46	-67.77	-69.15	-66.85	-65.11	-65.07	-65.07
0.00	0.00	0.00	48.52	6.54	-4.01	-16.18	-29.70	-38.68	-53.29	-58.66	-63.12	-66.54	-69.00	-69.71	-64.99	-65.04	-65.09	-65.09

VELOCITY COMPONENT IN X-DIRECTION

0.064	0.060	0.048	0.031	0.060	0.055	0.057	0.054	0.057	0.060	0.065	0.068	0.080	0.093	0.080	0.075	0.060	0.078	0.073
0.066	0.062	0.056	0.055	0.058	0.061	0.061	0.061	0.064	0.068	0.074	0.082	0.091	0.090	0.084	0.081	0.080	0.077	0.074
0.068	0.068	0.067	0.067	0.063	0.069	0.069	0.072	0.076	0.080	0.088	0.096	0.098	0.093	0.086	0.082	0.078	0.076	0.074
0.073	0.074	0.075	0.076	0.074	0.079	0.081	0.085	0.089	0.095	0.102	0.107	0.101	0.094	0.085	0.079	0.075	0.074	0.074
0.077	0.079	0.085	0.088	0.090	0.092	0.097	0.101	0.108	0.112	0.116	0.112	0.102	0.091	0.082	0.076	0.073	0.073	0.075
0.080	0.082	0.091	0.098	0.102	0.110	0.116	0.121	0.129	0.131	0.127	0.112	0.099	0.087	0.077	0.070	0.070	0.073	0.075
0.081	0.082	0.085	0.097	0.119	0.133	0.144	0.149	0.152	0.146	0.127	0.108	0.092	0.081	0.072	0.066	0.070	0.075	0.075
0.081	0.081	0.072	0.047	0.137	0.159	0.174	0.180	0.168	0.159	0.120	0.103	0.087	0.077	0.067	0.062	0.074	0.075	0.074
0.081	0.081	0.072	0.047	0.137	0.159	0.174	0.180	0.168	0.159	0.120	0.103	0.087	0.077	0.067	0.062	0.074	0.075	0.074

VELOCITY COMPONENT IN Y-DIRECTION

0.000	0.000	0.000	-0.031	-0.025	-0.032	-0.036	-0.046	-0.057	-0.069	-0.081	-0.102	-0.121	-0.129	-0.145	-0.161	-0.171	-0.167	-0.157
-0.001	-0.006	-0.017	-0.027	-0.028	-0.034	-0.041	-0.050	-0.061	-0.075	-0.091	-0.113	-0.126	-0.141	-0.157	-0.168	-0.172	-0.165	-0.159
-0.004	-0.012	-0.019	-0.020	-0.030	-0.037	-0.046	-0.056	-0.067	-0.082	-0.100	-0.121	-0.138	-0.155	-0.167	-0.173	-0.171	-0.164	-0.159
-0.004	-0.012	-0.019	-0.021	-0.034	-0.040	-0.049	-0.060	-0.073	-0.090	-0.111	-0.134	-0.152	-0.167	-0.173	-0.174	-0.168	-0.160	-0.161
-0.003	-0.012	-0.018	-0.024	-0.030	-0.041	-0.051	-0.065	-0.081	-0.101	-0.126	-0.152	-0.165	-0.175	-0.178	-0.174	-0.165	-0.159	-0.162
-0.002	-0.007	-0.012	-0.016	-0.029	-0.038	-0.051	-0.071	-0.091	-0.119	-0.148	-0.169	-0.178	-0.182	-0.180	-0.169	-0.158	-0.158	-0.162
-0.001	0.002	0.002	-0.010	-0.019	-0.034	-0.053	-0.079	-0.108	-0.144	-0.169	-0.180	-0.185	-0.184	-0.178	-0.160	-0.155	-0.161	-0.161
0.000	0.005	0.025	-0.022	0.003	-0.022	-0.055	-0.094	-0.128	-0.165	-0.184	-0.190	-0.190	-0.189	-0.175	-0.144	-0.159	-0.161	-0.160
0.000	0.000	0.000	0.053	0.021	-0.011	-0.050	-0.102	-0.134	-0.212	-0.196	-0.202	-0.200	-0.201	-0.181	-0.132	-0.158	-0.161	-0.160

PRESSURE COEFFICIENT = 1-(V/V2)**2

0.869	0.885	0.926	0.939	0.867	0.871	0.856	0.838	0.793	0.733	0.657	0.522	0.328	0.196	0.123	-0.008	-0.138	-0.084	0.038
0.862	0.876	0.891	0.861	0.869	0.846	0.827	0.799	0.750	0.669	0.562	0.379	0.233	0.110	-0.015	-0.104	-0.146	-0.055	0.021
0.850	0.848	0.844	0.842	0.844	0.803	0.780	0.735	0.674	0.581	0.438	0.244	0.083	-0.039	-0.130	-0.170	-0.134	-0.042	0.021
0.828	0.819	0.807	0.802	0.786	0.751	0.714	0.656	0.579	0.452	0.272	0.065	-0.067	-0.172	-0.186	-0.166	-0.083	0.006	-0.003
0.808	0.794	0.761	0.736	0.713	0.676	0.618	0.540	0.422	0.272	0.058	-0.132	-0.201	-0.244	-0.224	-0.150	-0.034	0.023	-0.015
0.796	0.783	0.731	0.683	0.639	0.558	0.465	0.376	0.200	0.002	-0.210	-0.316	-0.326	-0.301	-0.226	-0.068	0.042	0.031	-0.021
0.790	0.765	0.771	0.697	0.536	0.398	0.252	0.067	-0.108	-0.349	-0.429	-0.404	-0.362	-0.290	-0.181	0.040	0.078	-0.002	-0.007
0.791	0.790	0.815	0.913	0.401	0.179	-0.059	-0.313	-0.423	-0.894	-0.535	-0.492	-0.397	-0.327	-0.126	0.218	0.023	-0.001	0.011
0.791	0.791	0.835	0.839	0.368	0.132	-0.044	-0.366	-0.478	-1.244	-0.688	-0.643	-0.520	-0.477	-0.186	0.324	0.028	-0.003	0.011

NORMALIZED STATIC PRESSURE (P2 = .685)

0.959	0.964	0.977	0.961	0.958	0.959	0.955	0.949	0.935	0.916	0.892	0.850	0.788	0.747	0.724	0.683	0.641	0.659	0.697
0.956	0.961	0.966	0.963	0.959	0.951	0.946	0.937	0.921	0.896	0.862	0.804	0.758	0.719	0.680	0.657	0.639	0.668	0.692
0.953	0.952	0.951	0.950	0.951	0.938	0.931	0.916	0.897	0.868	0.823	0.762	0.711	0.673	0.644	0.631	0.643	0.672	0.692
0.946	0.943	0.939	0.938	0.932	0.921	0.910	0.891	0.867	0.828	0.771	0.705	0.664	0.631	0.627	0.633	0.659	0.687	0.684
0.940	0.935	0.925	0.917	0.910	0.898	0.880	0.855	0.818	0.771	0.703	0.643	0.622	0.608	0.614	0.638	0.674	0.692	0.680
0.936	0.932	0.915	0.900	0.886	0.864	0.838	0.803	0.748	0.686	0.619	0.585	0.582	0.590	0.614	0.664	0.698	0.695	0.678
0.934	0.932	0.928	0.905	0.854	0.811	0.764	0.713	0.651	0.575	0.550	0.558	0.571	0.594	0.628	0.698	0.710	0.684	0.683
0.934	0.934	0.942	0.973	0.811	0.742	0.666	0.586	0.552	0.403	0.516	0.530	0.560	0.582	0.645	0.754	0.692	0.685	0.689
0.934	0.934	0.948	0.949	0.807	0.745	0.671	0.570	0.534	0.293	0.468	0.482	0.521	0.535	0.626	0.787	0.694	0.684	0.689

BIBLIOGRAPHY

1. Wu, Chung-Hua, "A General Theroy of Three-Dimensional Flow in Subsonic and Supersonic Turbomachines of Axial-, Radial-, and Mixed-Flow Types," National Advisory Committee for Aeronautics, TN 2604, 1952.

2. Wang, S.Y., " A study on Numerical Methods for Computing Transonic Flows in Turbomachines," USAF-ASEE Summer Program Res. Rept. 1975.

3. Mach, K.D., "An Alternating-Direction-Implicity Solution of Subsonic Cascade Flow." USAF Aero Prop. Lab. AFAPL-TR-75-80, 1976.

4. Martin, J.C., "Finite Element Analysis of Fluid Flows." Proceedings of the 2nd Conference on Matrix Methods in Structural Mechanics, A.F.I.T. Wright-Patterson AFB, 1968.

5. Argyris, J.H., Mareczek, G. and Scharpf, D.W., "Two and Three Dimensional Flow Using Finite Elements," Aeronautical Journal of the Royal Aeronautical Society, Vol. 73, November, 1969, pp 961-964.

6. Norrie, D.H., and De Vries, G., "The Application of the Finite Element Technique to Potential Flow Problems," Trans. A.S.M.E., Journal of Applied Mechanics, Paper 71-APM-22.

7. Leonard, J.W., "Linearized Compressible Flow by the Finite Element Method," Bell Aerosystems Technical Note TCTN-2500-920156, 1969.

8. Gelder, D., "Solution of the Compressible Flow Equations," Int. J. Num. Methods in Engineering Vol. 3, pp. 35-43. 1971.

9. Olson, M.D., "Variational-Finite Element Mehtods for Two-Dimensional and Axisymetric Navier-Stokes Equations," in <u>Finite Element Methods in Flow Problems</u>, pp 103-106, UAH Press, 1974.

10. Baker, J.J., "Finite Element Solution Algorithm for Incompressible Fluid Dynamics," in <u>Finite Element Methods in Flow Problems</u>, pp 51-56. UAH Press, 1974.

11. Bratanow, T., Ecer, and Kobiske, "Finite Element Analysis of Unsteady Incompressible Flow Around an Oscillating Obstacle of Arbitrary Shape," AIAA Paper No. 73-91, 1973.

12. Chan, S.T.K. and Brashears, M.E., "Finite Element Analysis of Unsteady Transonic Flow," AIAA Paper No. 75-875, 1975.

13. Thompson, D.S., "Flow Through a Cascade of Airfoils," in Finite Element Methods in Flow Problems, pp. 707-720, UAH Press, 1974.

14. Hirsch, Ch., Warzee, G., "A Finite Element Method for Through-Flow Calculations in Turbomachines," A.S.M.E. 76-FE-12, 1976.

15. Chung, T.J., and Hooks, C.G., "Discontinuous Functions for Calculations in Transonic Flow," AIAA Peper no. 76-329, AIAA 9th Fluid and Plasma Dynamics Conference, July, 1976.

16. Delaney, R.A., and Young, D.F., "A Second-order Method of Characteristics for Two-Dimensional Unsteady Flow with Application to Turbomachinery Cascades," Ph.D. Dissertation, 1974.

17. Connor, J.J., and Brebbia, C.A., Finite Element Techniques for Fluid Flow, pp. 121-126, 1976.

APPENDIX A

DERIVATION OF LOCAL FINITE ELEMENT EQUATIONS

The potential flow through a cascade of blades of turbomachines can be governed by the Laplace equation, in terms of the stream function ψ below:

$$\psi_{,ii} = 0 \qquad (i = 1 + 2) \qquad \text{(A-1)}$$

Following the concept of the finite element method, this equation is assumed applicable to each element (a subdomain) of a selected finite element system for the boundary value problem considered, and the stream function, ψ, within each element is approximated by an interpolation function, such as:

$$\psi = \Omega_n \psi_n \qquad (n = 1, 2, \cdots N) \qquad \text{(A-2)}$$

where $\Omega_n(x,y)$ is generally referred to as interpolation functions, ψ_n is the value of the stream function at a node numbered 'n' of the element, and N is the total number of nodes for the element. One should note that the summation convention has been implied in all equations with repeated indices, unless indicated otherwise. According to the Method of Weighted Residuals, a residual for each element may be defined and minimized in the following:

$$R = \Psi_{,ii} \tag{A-3}$$

$$(R, \Omega_n) = \int_{A^e} \Psi_{,ii}\, \Omega_n\, dA = 0 \tag{A-4}$$

Applying the Green-Gauss theorem, equation (A-4) is converted into,

$$\int_S \Psi_{,i}\, n_i\, \bar{\Omega}_n\, dS - \int_{A^e} \Psi_{,i}\, \Omega_{n,i}\, dA = 0 \tag{A-5}$$

where S is a portion of the boundary where the Neumann boundary condition is prescribed. Substituting the interpolating functions, given in (A-2), into (A-5), one obtains the Local Finite Element Equations below:

$$A_{nm} \Psi_m = F_n \tag{A-6}$$

where:

$$A_{nm} = \int_{A^e} \Omega_{n,i}\, \Omega_{m,i}\, dA \tag{A-7}$$

$$F_n = \int_S \Psi_{,i}\, n_i\, \bar{\Omega}_n\, dS$$

APPENDIX B

THE INTERPOLATION FUNCTION FOR QRADRILATERAL ISOPARAMETRIC ELEMENT SYSTEM

Consider an arbitrarily shaped quadrilateral element as shown in Fig. 11. The isoparametric coordinates ξ and η whose values range from 0 to $\pm$ 1 are established at the centroid of the element. The reference cartesian coordinates X and Y and the variable u over the element are related to ξ and η by:

$$X = \alpha_q f_q(\xi,\eta) \qquad (q = 1, 2, 3, 4) \qquad \text{(B-1)}$$

$$Y = \alpha_q f_q(\xi,\eta) \qquad \text{"} \qquad \text{(B-2)}$$

$$u = \alpha_q f(\xi,\eta) \qquad \text{"} \qquad \text{(B-3)}$$

Take equation (B-3) for example, one may derive the interpolation functions as following:

$$\begin{aligned} u &= \alpha_q f_q(\xi,\eta) \\ &= \alpha_1 f_1(\xi,\eta) + \alpha_2 f_2(\xi,\eta) + \alpha_3 f_3(\xi,\eta) + \alpha_4 f_4(\xi,\eta) \\ &= [f_q(\xi,\eta)]\{\alpha_q\} \qquad \text{(B-4)} \end{aligned}$$

Using the four conditions at four corner nodes of the element, one can determine the values of four arbitrary constants $\{\alpha_q\}$,

$$
\begin{aligned}
u_i &= \alpha_1 f_1(\xi_i,\eta_i) + \alpha_2 f_2(\xi_i,\eta_i) + \alpha_3 f_3(\xi_i,\eta_i) + \alpha_4 f_4(\xi_i,\eta_i) \\
u_j &= \alpha_1 f_1(\xi_j,\eta_j) + \alpha_2 f_2(\xi_j,\eta_j) + \alpha_3 f_3(\xi_j,\eta_j) + \alpha_4 f_4(\xi_j,\eta_j) \\
u_k &= \alpha_1 f_1(\xi_k,\eta_k) + \alpha_2 f_2(\xi_k,\eta_k) + \alpha_3 f_3(\xi_k,\eta_k) + \alpha_4 f_4(\xi_k,\eta_k) \\
u_\ell &= \alpha_1 f_1(\xi_\ell,\eta_\ell) + \alpha_2 f_2(\xi_\ell,\eta_\ell) + \alpha_3 f_3(\xi_\ell,\eta_\ell) + \alpha_4 f_4(\xi_\ell,\eta_\ell)
\end{aligned}
\tag{B-5}
$$

or

$$
\begin{bmatrix} u_i \\ u_j \\ u_k \\ u_\ell \end{bmatrix} =
\begin{bmatrix}
f_1(\xi_i,\eta_i) & f_2(\xi_i,\eta_i) & f_3(\xi_i,\eta_i) & f_4(\xi_i,\eta_i) \\
f_1(\xi_j,\eta_j) & f_2(\xi_j,\eta_j) & f_3(\xi_j,\eta_j) & f_4(\xi_j,\eta_j) \\
f_1(\xi_k,\eta_k) & f_2(\xi_k,\eta_k) & f_3(\xi_k,\eta_k) & f_4(\xi_k,\eta_k) \\
f_1(\xi_\ell,\eta_\ell) & f_2(\xi_\ell,\eta_\ell) & f_3(\xi_\ell,\eta_\ell) & f_4(\xi_\ell,\eta_\ell)
\end{bmatrix}
\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}
\tag{B-6}
$$

or simple

$$\{u_N\} = [F_{Nq}]\{\alpha_q\} \qquad (N = i, j, k, \ell) \tag{B-6}$$

where

$$[F_{Nq}] = [f_q(\xi_N, \eta_N)]$$

from equation (B-6), one can solve for $\{\alpha_q\}$,

$$\{\alpha_q\} = [F_{Nq}]^{-1}\{u_N\} \tag{B-7}$$

Substituting (B-7) into (B-4), it yields,

$$u = [f_q(\xi,\eta)][F_{Nq}]^{-1}\{u_N\} \tag{B-8}$$

which is the general interpolation function of u.

APPENDIX C

BRIEF DESCRIPTION OF AN IMPROVED ASSEMBLING METHOD

To assemble the local finite element equation, one at each node of every element, into a global set of equations, the most straight forward way to do it is the so called Incidence Matrix Method. This scheme has been described in Chapter III. A disadvantage of this scheme is its requirement of a large amount of computer storage space to store a quite large incidence matrix (or symbol). Because of this, a lot of valuable computing time are needed to search for the coincidence between a local node of an element to a global node, and thus, makes this scheme not very practical especially for a computer without a very large memory. In view of this fact, an improved assembling method has been developed.

The basic idea of this new method is very simple. By carefully selecting the order of the global node number, the element number and the local element node number, the algebraic relationship among these three numbers can be established, such as:

$$IJ = F(NC,NR,NE,I) \tag{C-1}$$

where IJ is the global node numbers for nodes conected to node I, NC, NR and NE are column number, row number of global nodes and element number respectively. One should note that this equation is written in a general form for illustration of the basic idea. The detailed relations are built into the computer program listed in Appendix D. During the assembling process, the computer goes through the global nodes, one at a time, according to their order. At each global node, there is a global equation such as

$$A_{ij} \psi_j = F_i \tag{C-2}$$

where ψ_j are values of stream function at node j, A_{ij} are coefficients evaluated between node i and node j's, and F_i is a known value at node i. In general, j may vary from 1 to NT (the total number of global nodes), so that A_{ij}'s are usually evaluated NT times for each i using the general assmebling scheme such as the incidence matrix method. As mentioned previously, the coefficient matrix A_{ij} of the global equations is usually a band matrix due to the fact that only those j nodes directly connected to i node in a finite element (subdomain) affects node i, and thus, the corresponding A's are non-zero. The improved assembling method developed from the present study, determines which A_{ij} is non-zero, and then, this non-zero A_{ij} is assembled by the main program; Computing time is greatly saved as witnessed in Figure 8.

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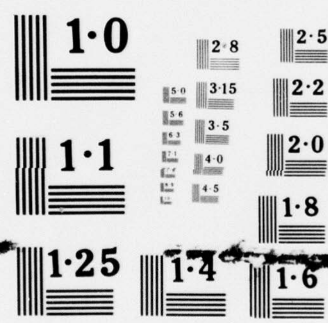

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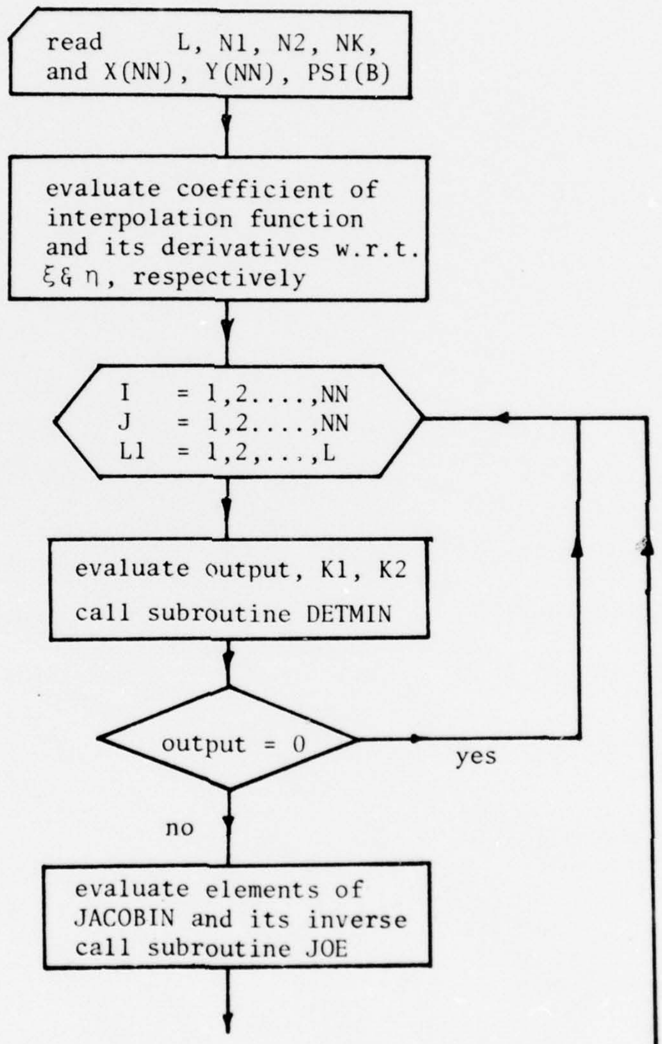
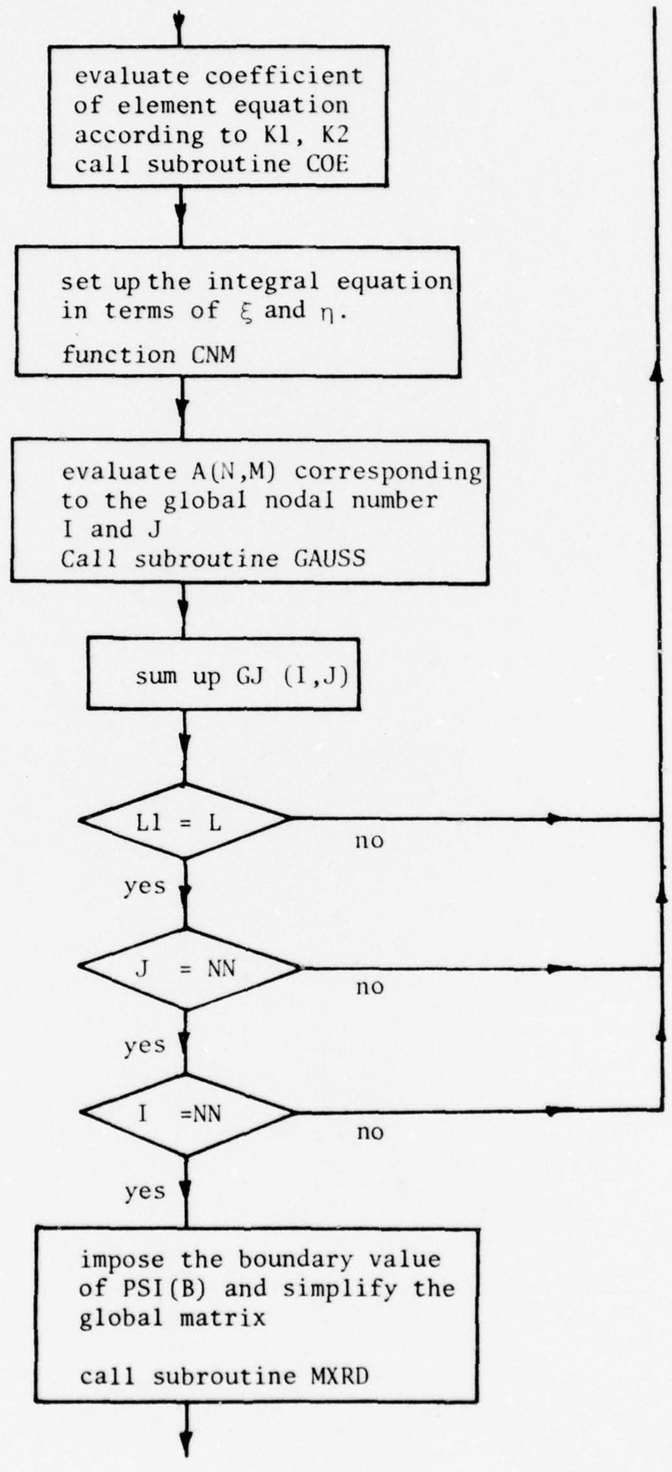
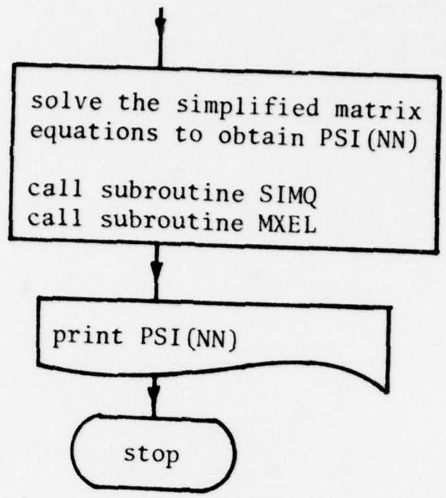
APPENDIX D

FLOW CHART OF COMPUTER PROGRAM

MAIN PROGRAM




evaluate coefficient of element equation according to K1, K2 call subroutine COE
set up the integral equation in terms of ξ and η.
function CNM
evaluate A(N,M) corresponding to the global nodal number I and J
Call subroutine GAUSS
sum up GJ (I,J)
L1 = L
no
yes
J = NN
no
yes
I =NN
no
yes
impose the boundary value of PSI(B) and simplify the global matrix
call subroutine MXRD


solve the simplified matrix equations to obtain PSI(NN)
call subroutine SIMQ
call subroutine MXEL
print PSI(NN)
stop

```
C *************************************************************************
C **                                                                     **
C **      TWO DIMENSIONAL FLOW THROUGH CASCADE WITH LINEAR               **
C **      QUADRILATERIAL ELEMENTS BY STREAMFUNCTION FORMULATION          **
C **                                                                     **
C *************************************************************************
C
C         L    :  NUMBER OF ELEMENTS
C         N1   :  NUMBER OF HORIZONTAL NODAL POINTS
C         N2   :  NUMBER OF VERTICAL NODAL POINTS
C         NK   : NUMBER OF NODES ON THE DIRICHLET TYPE BOUNDARY
C         NUN  :  NUMBER OF NODES ON THE NUMANN TYPE BOUNDARY
C         X(I):   X-COORDINATE OF NODE I
C         Y(I):   Y-COORDINATE OF NODE I
C         I1(I):  NODAL NUMBER ON THE DIRICHLET TYPE BOUNDARY
C         PSI(I):  BOUNDARY CONDITION AT NODAL NUMBER I ON THE DIRICHLET TYPE
C                 BOUNDARY
C         NN   :  NUMBER OF NODES ( =N1*N2 )
C         NUB  :  NUMBER OF UNKNOWN NODES ( =NN-NK )
C         X,Y COORDINATES READ IN FROM NODAL NUMBER 1 TO NODAL NUMBER NN
C         I1(I) READ IN FROM THE SMALLEST BOUNDARY NODAL NUMBER TO THE LARGEST
C                 BOUNDARY NODAL NUMBER
C      SUBROUTINE ABC  :  EVALUATE THE COEFFICIENTS OF THE INTERPOLATION
C               FUNCTION AND ITS DERIVATIVES W.R.T. ISOPARAMETRIC COORDINATES
C      SUBROUTINE JOE  :  CALCULATE THE COEFFICIENTS OF THE ENTRIES OF JACOBIAN
C      SUBROUTINE COE  :  CALCULATE THE COEFFICIENTS OF THE PRODUCTS OF INTER-
C               POLATION FUNCTIONS AND/OR ITS DERIVATIVES
C      FUNCTION CNM     :  SET UP THE INTEGRAL EQUATION IN TERMS OF ISOPARA-
C               METRIC COORDINATES
C      SUBROUTINE GAUSS1  :  INTEGRATE EQUATION CNM BY USING GAUSSIAN QUADRA-
C               TURE METHOD
C      SUBROUTINE AXRD  :  REDUCE N*N MATRIX AND N-VECTOR INTO (N-1)*(N-1)
C               MATRIX AND (N-1)-VECTOR AFTER IMPOSING ONE DIRICHLET TYPE
C                 BOUNDARY CONDITION
C      SUBROUTINE AXEL  :  ENLARGE (N-1)-VECTOR TO N-VECTOR AND REARRANGE IT
```

```
C              BY ADDING ONE DIRICHLET TYPE BOUNDARY CONDITION
C        SUBROUTINE SIMQ  :  SOLVE SIMULTANEOUS LINEAR EQUATIONS
C
         DIMENSION W(4,4),R1(4,2),R2(4,2),X(95),Y(95),S1(4),E1(4),
      1  C1(4),C2(4),EJ(2,2,2),B(4),E(5,9),GJ(95,95),
      2  GR(95),C(32,9),I1(38),PSI(95)
         EXTERNAL CHM
         COMMON R,B,EJ
         READ(2,100) L,N1,N2,NK,NUN
  100    FORMAT(1X,5I5)
         NN=N1*N2
         NUB=NN-NK
         READ(2,101) (X(I),Y(I),I=1,NN)
  101    FORMAT(1X,12F6.2)
         CALL WBC(W,N1,N2)
         DO 10 I=1,NN
         DO 10 J=1,NN
         GJ(I,J)=0.
         DO 20 L1=1,L
         CALL DETMIN(I,J,L1,N1,K1,K2,OUTPUT)
         IF(OUTPUT .EQ. 0.) GO TO 20
         CALL JOE(L1,N1,N2,X,Y,W1,W2,EJ,B)
         CALL COE(W,N1,N2,K1,K2)
         CALL GAUSS (CHM,-1.,1.,-1.,1.,2,2,CVAL)
         GJ(I,J)=GJ(I,J)+CVAL
   20    CONTINUE
   10    CONTINUE
C        *** TO SET UP GLOBAL EQUATIONS FOR UNKNOWN NODES ***
         READ(2,102) (I1(L1),L1=1,NK)
  102    FORMAT(1X,15I5)
         DO 40 L1=1,NK
         I1=I1(L1)
         READ(2,103) PSI(I1)
  103    FORMAT(1X,F7.4)
         LL=L1
```

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```
      CALL AXRB(II,NN,LL,PSI,GJ,GR)
 40   CONTINUE
      DO 11 I=1,NUB
      DO 11 J=1,NUB
      C(I+(NUB+NUM)*(J-1))=GJ(I,J)
 11   CONTINUE
C
C     *** SOLVE MATRIX ***
      CALL SIMQ(C,GR,NUB+NUM,KS)
      DO 61 L1=1,NN
      II=I1(L1)
      LL=L1
      CALL AXEL(II,NN,KK,LL,PSI,GR)
 61   CONTINUE
      DO 63 L1=1,NN
      PSI(L1)=GR(L1)
 63   CONTINUE
      WRITE(3,200) (PSI(L1),L1=1,NN)
200   FORMAT(1X,19F6.3)
      STOP
      END
```

```
      SUBROUTINE WBC (W,W1,W2)
      DIMENSION W(4,4),W1(4,2),W2(4,2),S1(4),E1(4)
     1,R(5,9),B(4),EJ(2,2,2)
      COMMON R,B,EJ
      INTEGER S1,E1
      DATA S1 /-1,1,1,-1/
      DATA E1 /-1,-1,1,1/
      DO 1 I=1,4
      W(I,1)=1./4.
      W(I,2)=S1(I)/4.
      W(I,3)=E1(I)/4.
      W(I,4)=S1(I)*E1(I)/4.
      W1(I,1)=S1(I)/4.
      W1(I,2)=S1(I)*E1(I)/4.
      W2(I,1)=E1(I)/4.
      W2(I,2)=E1(I)*S1(I)/4.
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE [illegible](I,J,L1,M1,K1,K2,OUTPUT)
      DIMENSION GN(4)
      INTEGER GN
      COMMON F,B,EJ
      NR=(L1-1)/(M1-1)
      NC=L1-NR*(M1-1)
      GN(4)=NR*M1+NC
      GN(3)=GN(4)+1
      GN(2)=GN(3)+N1
      GN(1)=GN(2)-1
      IF( I .EQ. GN(1) .OR. I .EQ. GN(2) .OR. I .EQ. GN(3) .OR. I .EQ.
     1 GN(4) ) GO TO  40
      GO TO 30
40    IF( J .EQ. GN(1) .OR. J .EQ. GN(2) .OR. J .EQ. GN(3) .OR. J .EQ.
     1 GN(4) ) GO TO 20
30    OUTPUT=0.
      GO TO 60
20    DO 50 K=1,4
      IF( I .EQ. GN(K) )GO TO 55
50    CONTINUE
55    K1=K
      DO 56 K=1,4
      IF( J .EQ. GN(K) ) GO TO 57
56    CONTINUE
57    K2=K
      OUTPUT=1.
60    RETURN
      END
```

```
      SUBROUTINE DELEMT(M1,M2,DET)
      INTEGER M1,M2,TH1,TH
      DIMENSION DET(72,4,95),TH(4),R(5,9),B(4),EJ(2,2,2)
      COMMON R,B,EJ
      DO 3 L=1,M2-1
      DO 3 M=1,M1-1
      I=M+M1*(L-1)
      N=M+(M1-1)*(L-1)
      TH(1)=I+M1
      TH(2)=I+M1+1
      TH(3)=I+1
      TH(4)=I
      DO 3 J=1,4
      TH1=TH(J)
      DET(N,J,TH1)=1.
3     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE JOE (II,N1,N2,X,Y,W1,W2,E,BB)
      DIMENSION W1(4,2),W2(4,2),X(95),Y(95),C1(4),C2(4),BB(4),
    1 E(2,2,2),R(5,9),B(4),EJ(2,2,2)
      COMMON R,B,EJ
      INTEGER C1,C2,AN1,AN3,I1,M
      DATA C1 /0,1,1,0/
      DATA C2 /1,1,0,0/
      AN1=(I1-1)/(N1-1)
      AN3=I1-AN1*(N1-1)
      II=AN1*N1+AN3
      DO 7 I=1,2
      DO 7 J=1,2
      E(I,J,1)=0.
      E(I,J,2)=0.
      DO 7 K=1,4
      M=C1(K)+C2(K)*N1+II
      IF (I .GT. 1) GO TO 1
      A1=W1(K,1)
      A2=W1(K,2)
      GO TO 2
    1 A1=W2(K,1)
      A2=W2(K,2)
    2 IF (J .GT. 1) GO TO 3
      B1=X(M)
      GO TO 4
    3 B1=Y(M)
    4 E(I,J,1)=E(I,J,1)+A1*B1
      E(I,J,2)=E(I,J,2)+A2*B1
    7 CONTINUE
      BB(1)=E(1,1,1)*E(2,2,1)-E(1,2,1)*E(2,1,1)
      BB(2)=E(1,1,1)*E(2,2,2)-E(1,2,1)*E(2,1,2)
      BB(3)=E(1,1,2)*E(2,2,1)-E(1,2,2)*E(2,1,1)
      BB(4)=E(1,1,2)*E(2,2,2)-E(1,2,2)*E(2,1,2)
      RETURN
      END
```

```
      SUBROUTINE COE(W,W1,W2,I,J)
      DIMENSION W(4,4),W1(4,2),W2(4,2),R(5,9),P(4),EJ(2,2,2)
      COMMON R,P,EJ
      R(1,1)=W1(I,1)*W1(J,1)
      R(1,2)=W1(I,1)*W1(J,2)+W1(J,1)*W1(I,2)
      R(1,3)=W1(I,2)*W1(J,2)
      R(2,1)=W1(I,1)*W2(J,1)
      R(2,2)=W1(I,1)*W2(J,2)
      R(2,3)=W2(J,1)*W1(I,2)
      R(2,4)=W1(I,2)*W2(J,2)
      R(3,1)=W2(I,1)*W1(J,1)
      R(3,2)=W2(I,2)*W1(J,1)
      R(3,3)=W2(I,1)*W1(J,2)
      R(3,4)=W2(I,2)*W1(J,2)
      R(4,1)=W2(I,1)*W2(J,1)
      R(4,2)=W2(I,1)*W2(J,2)+W2(I,2)*W2(J,1)
      R(4,3)=W2(I,2)*W2(J,2)
      R(5,1)=W(I,1)*W(J,1)
      R(5,2)=W(I,2)*W(J,1)+W(I,1)*W(J,2)
      R(5,3)=W(I,3)*W(J,1)+W(I,1)*W(J,3)
      R(5,4)=W(I,4)*W(J,1)+W(I,3)*W(J,2)+W(I,2)*W(J,3)+W(I,1)*
     1       W(J,4)
      R(5,5)=W(I,2)*W(J,2)
      R(5,6)=W(I,4)*W(J,2)+W(I,2)*W(J,4)
      R(5,7)=W(I,3)*W(J,3)
      R(5,8)=W(I,4)*W(J,3)+W(I,3)*W(J,4)
      R(5,9)=W(I,4)*W(J,4)
      RETURN
      END
```

```
      SUBROUTINE GAUSS (CF,XL,XU,YL,YU,NOX,NOY,CVAL)
      DIMENSION Q1(52),Q2(24),Q3(32),NO(8),NS(8),QG(108)
     1 ,R(5,9),B(4),EJ(2,2,2)
      EQUIVALENCE (Q1(1),QG(1)),(Q2(1),QG(53)),(Q3(1),QG(77))
        COMMON R,B,EJ
      DATA Q1
     $             /.2886751345948128822E0,0.5E0,.4305681557970262 9E0,
     $.1739274225687269 3E0,.1699905217924281 3E0,.3260725774312730 7E0,
     $ 0.4801449282487681 2E0,.5061426814518813 0E-1,.3983332387068133 7E0,
     $.1111905172266087 24E0,.2627662049581644 9E0,.1568533229389436 4E0,
     $.9171732124782490E-1,.1813418916891809 9E0,.4869532642585858 6E0,
     $.3333567215434407E-1,.4325316833444922 5E0,.7472567457529 03E-1,
     $.3397047841496122E0,.1095431812579910 2E0,.2166976970646236E0,
     $.1346333591549981 8E0,.7443716949081560 5E-1,.1477621123573764 4E0,
     $0.4907803171233596 3E0,.2358766819325591 4E-1,.4520586281852374 3E0,
     $.5346966299765921 5E-1,.3849513370971523 4E0,.8003916427167311E-1,
     $.2936589771433087 2E0,.1015837133615329 6E0,.1839157494990901 0E0,
     $.1167462682691774 0E0,.6261670425573445 8E-1,.1245735229067013 9E0,
     $.4947004674958249 7E0,.1357622970587704 7E-1,.4722875115366162 9E0,
     $.3112676196932394 6E-1,.4328156011939158 7E0,.4757925584124639 2E-1,
     $.3777022041775015 2E0,.6231448562776693 6E-1,.3089381222013218 7E0,
     $.7479799440828837E-1,.2290083888286136 9E0,.8457825969750127E-1,
     $.1408017753896294 6E0,.9130170752246179E-1,.4750625491881872 0E-1,
     $.9472530522753425E-1 /
      DATA Q2
     $             / 0.4975936099985106 8E+0    ,0.6170614899993599 8E-2 ,
     *             0.4873642779856547 5E+0   ,   0.1426569431446683 2E-1 ,
     *             0.4691372760013663 8E+0   ,   0.2213871940870990 3E-1 ,
     *             0.4432077635022005 2E+0   ,   0.2964929245771889 0E-1 ,
     *             0.4100009929869514 6E+0   ,   0.3667324070554015 3E-1 ,
     *             0.3700620957892771 8E+0   ,   0.4309508076597663 8E-1 ,
     *             0.3240468259684877 8E+0   ,   0.4880932605205694 4E-1 ,
     *             0.2727107356944197 7E+0   ,   0.5372213505798281 7E-1 ,
     *             0.2168967538130225 7E+0   ,   0.5775283402686280 1E-1 ,
     *             0.1575213398480816 9E+0   ,   0.6083523646390169 6E-1 ,
```

```
     *            0.95559433736808150E-1    ,    0.62918728173414148E-1 ,
     *            0.32028464313028134E-1    ,    0.63969097673376078E-1 /
      DATA Q3
     $             /0.49863193092474078E+0 ,    0.35093050047350483E-2 ,
     *            0.49280575577263417E+0    ,    0.81371973654528350E-2 ,
     *            0.48238112779375322E+0    ,    0.12696032654631030E-1 ,
     *            0.46745303796886984E+0    ,    0.17136931456510717E-1 ,
     *            0.44816057788302606E+0    ,    0.21417949011113340E-1 ,
     *            0.42468380686628499E+0    ,    0.25499029631188088E-1 ,
     *            0.39724189798397120E+0    ,    0.29342046739267774E-1 ,
     *            0.36609105937014484E+0    ,    0.32911111388180923E-1 ,
     *            0.33152213346510760E+0    ,    0.36172897054424253E-1 ,
     *            0.29385787862038116E+0    ,    0.39096947893535153E-1 ,
     *            0.25344995446611470E+0    ,    0.41655962113473378E-1 ,
     *            0.21067563806531767E+0    ,    0.43826046502201906E-1 ,
     *            0.16593430114106382E+0    ,    0.45586939347881942E-1 ,
     *            0.11964368112606854E+0    ,    0.46922199540402283E-1 ,
     *            0.72235980791398250E-1    ,    0.47819360039637430E-1 ,
     *            0.24153832843869158E-1    ,    0.48270044257363900E-1 /
     $,NQ/2,4,8,10,12,16,24,32/,
     $ NS/1,3,7,15,25,37,53,77/
      DO 400 L=1,8
      IF(NQX.EQ.NQ(L)) GO TO 401
400   CONTINUE
905   FORMAT('0 CALLING PARAMETER =',I5,' INTEGRATION NOT POSSIBLE'//)
      WRITE(3,905) NQX
      RETURN
401   CONTINUE
      NP=NS(L)
      NE=NP+NQX-1
      DO 500 M=1,8
      IF(NQY.EQ.NQ(M)) GO TO 501
500   CONTINUE
      WRITE(3,905) NQY
      RETURN
```

```
501   CONTINUE
      NP=NS(M)
      NE=NP+NCY-1
      AX=0.5*(XU+XL)
      BX=XU-XL
      CVAL=0.
      AY=0.5*(YU+YL)
      BY=YU-YL
      DO 200 J=NP,NE,2
      DX=QG(J)*BX
      APD=AX+DX
      AMD=AX-DX
      CY1=0.
      CY2=0.
      DO 100 K=NP,NE,2
      DY=QG(K)*BY
      CY1=CY1+QG(K+1)*(CF(APD,AY+DY)+CF(APD,AY-DY))
      CY2=CY2+QG(K+1)*(CF(AMD,AY+DY)+CF(AMD,AY-DY))
100   CONTINUE
      CVAL=CVAL+QG(J+1)*BY*(CY1+CY2)
200   CONTINUE
      CVAL=BX*CVAL
      RETURN
      END
```

```
      FUNCTION CNM(S,E)
      DIMENSION EJ(2,2,2),B(4),R(5,9)
      COMMON R,B,EJ
      BF=B(1)+B(2)*S+B(3)*E+B(4)*S*E
      EJI11=(EJ(2,2,1)+EJ(2,2,2)*S)/BF
      EJI21=-(EJ(2,1,1)+EJ(2,1,2)*S)/BF
      EJI12=-(EJ(1,2,1)+EJ(1,2,2)*E)/BF
      EJI22=(EJ(1,1,1)+EJ(1,1,2)*E)/BF
      F1=(R(1,1)+R(1,2)*E+R(1,3)*E**2)*(EJI21**2+
     1      EJI11**2)
      F2=(R(2,1)+R(2,2)*S+R(2,3)*E+R(2,4)*E*S+R(3,1)+
     1      R(3,2)*S+R(3,3)*E+R(3,4)*S*E)*(EJI11*EJI12+
     2      EJI21*EJI22)
      F3=(R(4,1)+R(4,2)*S+R(4,3)*S*S)*(EJI12**2+
     1      EJI22**2)
      CNM=(F1+F2+F3)*BF
      RETURN
      END
```

```
      SUBROUTINE MXRD ( II,NN,L,PSI,A,BB )
C     THIS SUBROUTINE REDUCES NN*NN MATRIX TO (NN-1)*(NN-1) MATRIX
      DIMENSION A(95,95),BB(95),PSI(1),R(5,9),B(4),EJ(2,2,2)
      COMMON R,B,EJ
      INTEGER II,NN
      N=NN-(L-1)
      MN=II-(L-1)
      DO 6 I=1,N
      IF(I .GT. MN) GO TO 3
      IF(I .EQ. MN) GO TO 6
      BB(I)=BB(I)-A(I,MN)*PSI(II)
      DO 2 J=1,N
      IF(J .GT. MN) GO TO 1
      IF(J .EQ. MN) GO TO 2
      A(I,J)=A(I,J)
      GO TO 2
    1 A(I,J-1)=A(I,J)
    2 CONTINUE
      GO TO 6
    3 BB(I-1)=BB(I)-A(I,MN)*PSI(II)
      DO 5 J=1,N
      IF(J .GT. MN) GO TO 4
      IF(J .EQ. MN) GO TO 5
      A(I-1,J)=A(I,J)
      GO TO 5
    4 A(I-1,J-1)=A(I,J)
    5 CONTINUE
    6 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AXEL ( II,NN,NK,L,PSI,BB )
C     THIS SUBROUTINE EXPANDS (NN-NK)*(NN-NK) MATRIX TO (NN-NK+1)*(NN-NK+1)
C     MATRIX
      DIMENSION PSI(1),BB(95),F(5,9),B(4),EJ(2,2,2),B1(95)
      COMMON F,B,EJ
      DO 6 I=1,NN-NK+(L-1)
      IF(I .GE. II) GO TO 3
      B1(I)=BB(I)
      GO TO 6
    3 B1(I+1)=BB(I)
    6 CONTINUE
      B1(II)=PSI(II)
      DO 9 I=1,NN-NK+L
      BB(I)=B1(I)
    9 CONTINUE
      RETURN
      END
```

BIOGRAPHICAL SKETCH OF THE AUTHOR

Tsu-Yi Su was born in Taipei, Taiwan, Republic of China, on September 2,1951, the son of Wei-Tsean and Tze-Ching Su.

After graduating from the High School of National Taiwan Normal University, in 1970, he entered the Tatung Institute of Technology from which he received the degree of Bachelor of Science in Mechanical Engineering in 1974.

From 1974 to 1975 he was employed by Sinotech Engineering consultants Inc. as a junior engineer.

From 1975 to 1978 he has taken graduate courses toward the Master of Science degree in Mechanical Engineering at the University of Mississippi. He worked for the University of Mississippi since 1975 as a graduate research assistant.

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ABSTRACT

COMPUTER SIMULATION OF TWO-DIMENSIONAL CASCADE FLOWS OF IDEAL FLUIDS

SU,TSU-YI B.S., Tatung Institute of Technology, 1974.
M.S., University of Mississippi, 1978.
Thesis Directed by Dr. Shu-Yi Wang, Associate Professor of Mechanical Engineering.

The steady two-dimensional flow of an inviscid and incompressible fluid is simulated by the digital computer using the finite element method. The finite element method is chosen not only because it is simple and general, but also because it is extremely suitable for the complex geometry of the flow environment. The computer program developed for the simulation contains several subroutines. Each of them is used to carry out a specific step in the model formulation as well as solution procedures. All subroutines are designed in a general form, so that they can be used to simulate models of higher level of sophistication with little or no modification. The potential cascade flow results obtained from the present study are in good agreement with those of others published.